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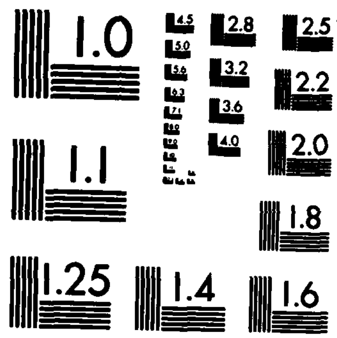

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Waveguide theory is applied to the interpretation of data on reflection and propagation of sound in a water-filled impedance tube and in an acoustic calibrator. Computer programs are described and documented that: (1) relate propagation speed and attenuation for the sample in the impedance tube to the complex reflection coefficient; (2) relate propagation speed in the sample in the impedance tube to the elastic moduli of the sample material, both real and complex; and (3) compute the propagation speed in the acoustic calibrator.
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Item 20 continued.

A correction factor is developed to account for the lack of rigidity of the reflector in the impedance tube. The major conclusion with respect to the data reduction for the impedance tube is the observation that the relevant wave speed is not necessarily the dilatational wave speed, but is a function of the space between sample and tube wall. By modifying the experimental arrangement one may measure two separate elastic moduli.

CONTENTS

COMPUTATION OF PROPAGATION SPEED AND REFLECTION OF AXIALLY SYMMETRIC WAVES IN COMPOSITE CYLINDERS, WITH APPLICATION TO IMPEDANCE TUBE AND CALIBRATOR

1.0. INTRODUCTION

Various instruments used at the Underwater Sound Reference Detachment of the Naval Research Laboratory (NRL-USRD) are based on acoustic waveguide technology. Examples are the impedance tube [1], inertial calibrators [2,3], and the Low-Frequency Facility's systems [4,5]. A common feature of these instruments is the creation of an acoustic field with well-defined properties in a limited space, as contrasted with free-field measurements. The field description of this class of instruments is based on waveguide theory. In this report an analysis of the waveguide aspects of these instruments is presented. Discussion and documentation of the computer programs used in the data reduction are given. The correspondence between predictions from the analysis and the data will be a measure of the extent to which simple waveguide theory is adequate in describing the phenomena.

Waveguide theory is well established and described [6,7]. Some of its features are given in this report to serve as a basis for the understanding of the computer programs and quantitative conclusions. It is assumed that the wave propagation takes place in one dimension only. The field perpendicular to this direction is spatially limited by shape and properties of physical bodies. This does not exclude the possibility that particle velocity and pressure may be functions of more than one coordinate. It will be assumed, though, that at interfaces perpendicular to the direction of propagation, the boundary conditions are only imposed on averaged axial velocity and averaged pressure. Usually, one applies waveguide theory to cases where the cross-sectional dimension is small compared with the wavelength; a consequence is that the radial variation of the field variables is small. The analysis developed in this report does not account for the influence of finite cylinder length and its attending termination on the wave pattern. This influence will be less the larger the ratio of length to diameter of the various cylindrical elements of the system.

The hierarchy of computer programs documented here may be understood by reference to the sketches of impedance tube, Fig. 1, and free surface calibrator, Fig. 2. For the cylindrical sample in the impedance tube, Fig. 1, one wants to evaluate the propagation speed in the axial direction given the values of a pair of elastic moduli of the solid material. In general, propagation speed and elastic moduli are complex, accounting for the existence of dissipation of acoustic energy in the material. The reflection of a plane wave from the front face of the sample may then be computed. The propagation speed in the water will be influenced by the lack of rigidity of the cylinder wall. This problem is more serious in the case of the calibrator where the

wall thickness is considerably less than that of the impedance tube. This influence is numerically determined. The sequence of computations in data analysis of the impedance tube is just the opposite of the one sketched. One experimentally determines the complex reflection coefficient. From this a complex propagation speed is inferred and, in turn, this propagation speed is related to the elastic moduli of the solid material. This inverse procedure may be effected by a combination of root-search techniques and the programs presented here.

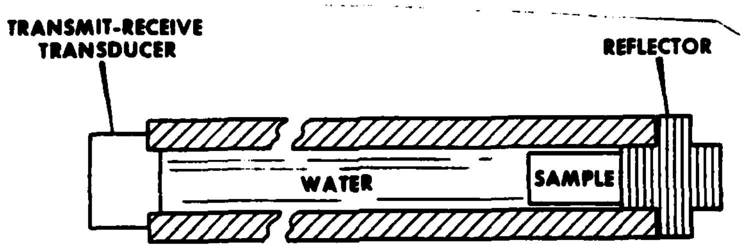

Fig. 1 - Section of acoustic impedance tube.

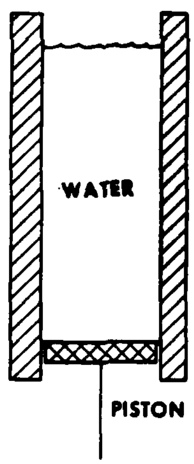

Fig. 2 - Principle of free surface acoustic calibration device.

In this report three computer programs are described and documented. Program RTUBE computes the complex propagation speed in the sample in the impedance tube from the measured complex reflection coefficient. Program WGUIDE, with various options, determines real propagation speed in the sample in the impedance tube and in the water in the calibrator from a pair of real elastic moduli characterizing the material of the sample and tube wall, respectively. Program IMPED computes the complex wave speed from two complex elastic moduli for the sample in the impedance tube.

In the next section the theory for reflection of the signal in the impedance tube is given. Then the theory of composite waveguides is developed and the various dispersion relations are derived applicable to the boundary conditions for impedance tube and calibrator. Examples are given for wave propagation in samples of steel and rubber in the impedance tube, and for the influence of the wall of the calibrator. The detailed discussion and documentation of the computer programs is given in Appendix A. Appendix B deals with the application of a correction factor to the measurement in the impedance tube that accounts for the lack of perfection of the reflector.

An important conclusion of this study is the observation that the sound speed in the sample in an impedance tube, ignoring end effects, is not necessarily the dilatational wave speed but a function of the size of the fluid-filled gap between sample and tube wall, and the elastic properties of the sample material. This effect may be utilized to obtain more information on the elastic moduli by varying the sample radius, and also by not admitting fluid to fill this gap.

2.0. REFLECTION COEFFICIENT IN IMPEDANCE TUBE

It is assumed that a wave propagates in the axial direction identified with the z coordinate and that the wave pattern is periodic in z. Thus the field variables will contain a factor exp $i(\omega t - kz)$, where ω is the angular frequency and k the wave number; the phase speed c is related by $\omega = kc$. Because of the geometry, it is assumed that the field variables do not depend on the azimuth angle. The pressure and axial particle velocity are, in general, functions of the radial distance r. For the sake of the computation of the reflection coefficient, it is assumed that one may represent the wave as a plane wave with average values for pressure and velocity. The sample of length d is backed by a reflector with acoustic impedance Z_r. One can show [8] that the input impedance at the front end of the sample is

$$Z = (\rho_s c/S) \frac{Z_r + i(\rho_s c/S) \tan kd}{(\rho_s c/S) + i Z_r \tan kd}, \tag{1}$$

where ρ_s is the density of the sample
c is the sound speed in the sample
S is the cross-sectional area
k is the wave number in the sample.

If one applies this same formula to the reflector behind the sample, where the impedance of the air backing the reflector may be ignored, one sees that the impedance of the reflector, Z_r, is given by $Z_r = i(\rho c)_r \tan k_r \ell$, where the subscript r refers to the reflector, and ℓ is the length of the reflector. In practice, one chooses ℓ such that $k\ell = \pi/2$ for some intermediate frequency and then the impedance Z_r is virtually infinite for a reasonably large bandwidth.

The reflection coefficient (complex) r of the sample is defined as the ratio of the amplitude of the reflected to that of the incoming wave. Equating impedances at the interface leads to the relation between r and the wave number k according to

$$\frac{Z}{(\rho_o c_o/S)} = \frac{1 + r}{1 - r} = -i(\rho_s/\rho_o)\ (k_o d/kd)\ \cot(kd) \quad , \tag{2}$$

where ρ_o is the density of and k_o the wave number in the fluid. The program RTUBE determines the complex root kd of this equation from the measured complex value of r.

The data reduction incorporated in program RTUBE is based on Eq. (2) which is derived under the assumption that the reflector is ideal; i.e., that the amplitude and phase of the reflected pressure wave are the same as those of the incident wave. To account for the lack of rigidity of the reflector, and to avoid measurement of absolute magnitude and phase, the wave reflected in the tube with the sample removed is measured. The reflection coefficient r is computed according to $r = (r_{sa}/r_{st})\exp(-2ik_o d)$, where r_{sa} is the reflection with the sample in place, and r_{st} is the reflection without the sample. The exponential factor accounts for the fact that the sample is replaced by a column of water with a given phase shift. This r is only approximately correct; an additional correction factor is described in Appendix B.

3.0. THEORY OF WAVE PROPAGATION IN INFINITE CYLINDERS AND CYLINDRICAL SHELLS

In an elastic solid of infinite extent there are two different types of wave motion. One is an irrotational wave, usually denoted as a dilatational wave, and the other is an incompressible motion, or shear wave. Mathematically, they can be described by a scalar and vector potential ϕ and $\vec{A}$ respectively. In the axially symmetric situation of waveguides, only the azimuthal component of the vector potential is needed, henceforth indicated by the function H(r,z).

The general relations used below can be found in Ref. 9. The particle displacement $\vec{u}$ is given in terms of the potentials by

$$\begin{aligned} u_r &= \frac{\partial\phi}{\partial r} - \frac{\partial H}{\partial z} \\ u_z &= \frac{\partial\phi}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}(rH) \ . \end{aligned} \tag{3}$$

The stresses pertinent to the analysis for the case of axial symmetry are given in terms of the strains by

$$\begin{aligned} \tau_{rr} &= \lambda e + 2\mu e_{rr} \ , \\ \tau_{zz} &= \lambda e + 2\mu e_{zz} \ , \\ \tau_{\theta\theta} &= \lambda e + 2\mu e_{\theta\theta} \ , \end{aligned} \tag{4}$$

$$\tau_{rz} = 2\mu e_{rz} \ ,$$

where

$$e_{rr} = \frac{\partial u_r}{\partial r} \ ,$$

$$e_{\theta\theta} = \frac{u_r}{r} \ ,$$

$$e_{zz} = \frac{\partial u_z}{\partial z} \ ,$$

$$e_{rz} = 1/2\left(\frac{\partial u_z}{\partial r} + \frac{\partial u_r}{\partial z}\right) \ ,$$

$$e = e_{rr} + e_{\theta\theta} + e_{zz} \ ,$$

and λ and μ are Lamé constants.

The equations of motion are

$$\frac{\partial \tau_{rr}}{\partial r} + \frac{\partial \tau_{rz}}{\partial z} + \frac{\tau_{rr} - \tau_{\theta\theta}}{r} = \rho_s \frac{\partial^2 u_r}{\partial t^2} \ ,$$

$$\frac{\partial \tau_{rz}}{\partial r} + \frac{\partial \tau_{zz}}{\partial z} + \frac{1}{r}\tau_{rz} = \rho_s \frac{\partial^2 u_z}{\partial t^2} \ . \tag{5}$$

By combining Eqs. (3) through (5) one derives the following wave equations for the potentials ϕ and H

$$c_d{}^2 \Delta\phi = \frac{\partial^2 \phi}{\partial t^2} \tag{6}$$

and

$$c_s{}^2 \ (\Delta H - H/r^2) = \frac{\partial^2 H}{\partial t^2} \ , \tag{7}$$

where c_d is the wave speed for dilatational waves, $c_d{}^2 = (\lambda + 2\mu)/\rho_s$, c_s is the wave speed for shear waves, and $c_s{}^2 = \mu/\rho_s$; Δ is the Laplacean

operator for cylindrical coordinates $\Delta = \frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial}{\partial r}) + \frac{\partial^2}{\partial z^2}$.

If one assumes solutions for ϕ and H in the form of a wave traveling in the z direction, according to

$$\phi = f(r)\, e^{i(\omega t - kz)}$$

and (8)

$$H = g(r)\, e^{i(\omega t - kz)} ,$$

the functions f(r) and g(r) are solutions of

$$f'' + f'/r - q^2 f = 0 \tag{9}$$

and

$$g'' + g'/r - (s^2 + {}^1/r^2)g = 0 , \tag{10}$$

where $q^2 = k^2 - k_d^2$ and $s^2 = k^2 - k_s^2$, k_d and k_s are the wave numbers for dilatational and shear waves. In terms of the variables q' and s', defined by $(q')^2 = -q^2$ and $(s')^2 = -s^2$, the general solutions to Eqs. (9) and (10) are $f = A\, J_o\, (q'r) + B\, Y_o\, (q'r)$ and $g = C\, J_1(s'r) + D\, Y_1(s'r)$. J_n is the Bessel function of the first kind with order n, and Y_n is the Bessel function of the second kind with order n.

The compressional wave in the fluid is represented by a velocity potential ϕ_o which is a solution of the wave equation

$$c_o{}^2 \Delta\phi_o = \frac{\partial^2\phi_o}{\partial t^2} , \tag{11}$$

where c_o is the wave speed in the fluid.

The velocity components U_r and U_z of the particle velocity in the fluid follow from $U_r = \frac{\partial\phi_o}{\partial r}$ and $U_z = \frac{\partial\phi_o}{\partial z}$. The solution of Eq. (11) representing a traveling wave in the z direction is

$$\phi_o = [E\, J_o\, (q_o'r) + F\, Y_o\, (q_o'r)]\, e^{i(\omega t - kz)},$$

where $(q_o')^2 = k_o{}^2 - k^2$, k_o is the wave number of the compressional wave in the fluid. Application of the boundary conditions leads to the dispersion relations for the phase speed c. For a wave in an infinite cylinder in

vacuum, the analysis has been performed by Pochhammer and Chree [10]. A comprehensive set of calculations of the phase speed based on the Pochhammer-Chree solution were performed by Bancroft [11].

4.0. PROPAGATION SPEED OF AXIALLY SYMMETRIC WAVES IN COMPOSITE CYLINDERS

The determination of phase speed in composite cylinders is accomplished by imposing the pertinent boundary conditions on the stresses and particle velocities derived in Section 3. An overview of the various optior in the computer program is presented in Fig. 3.

The physical situation in the impedance tube is represented ir Option 1. A solid cylinder is surrounded by a thin layer of fluid d the center wall of the tube is assumed to be rigid. For comparison, a outer boundary is considered in Option 2, although this case can hardly ι .alized in practice. The calibrator situation is represented by Option 3, where the influence of a nonrigid wall on the fluid wave speed is evaluated. Options 4 and 5 reflect the situation where a soft coating is applied to the inside of the calibration tube in order to slow down the wave in the liquid; in Option 4 the coating is cemented to the wall, in Option 5 it is free to move tangentially at the wall. In Option 6 the wave speed in a fluid cylinder with rigid boundary is computed, in Option 7 the wave speed in a fluid cylinder with free boundary is computed.

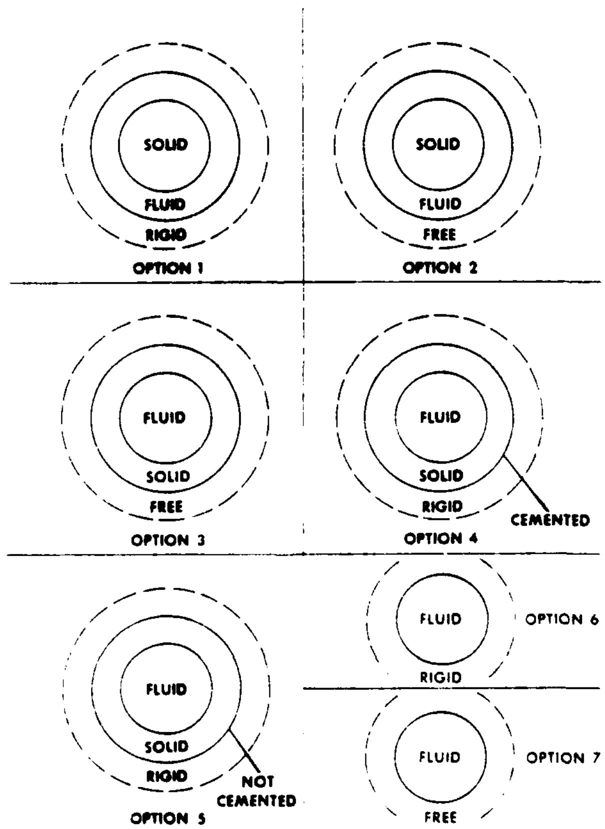

Fig. 3 - Cross section of waveguides in situations corresponding to options in program WGUIDE.

In the real variable program WGUIDE, the variables q', s', and q_o' may be either real or imaginary. In order to avoid the necessity of complex calculations, the transformations $q' \rightarrow iq$; $s' \rightarrow is$, and $q_o' \rightarrow iq$ are applied whenever imaginary values appear. The Bessel functions are transformed according to the identities $J_o(ix) = I_o(x)$, $J_1(ix) = iI_1(x)$, $Y_o(ix) = -K_o(x)$, and $Y_1(ix) = iK_1(x)$. $I_n(x)$ and $K_n(x)$ are modified Bessel functions [12].

4.1. Wave Speed in Sample in Impedance Tube

The following set of boundary conditions is applied to the field variables in the solid cylinder and surrounding fluid if the outer wall is rigid (Option 1).

a. The normal stress τ_{rr} in the solid is equal to the negative of the pressure p in the fluid at the surface r = a, where a is the radius of the cylinder. The pressure follows from

$$\frac{\partial p}{\partial r} = -\rho_o \frac{\partial U}{\partial t} . \quad \text{Thus } p = -\rho_o \frac{\partial \phi_o}{\partial t} \text{ at } r = a.$$

b. The tangential stress τ_{rz} is zero in the solid for r = a.

c. The normal component of the particle velocity is continuous at the interface between solid and fluid at r = a.

d. The normal component of the particle velocity in the fluid U_r is zero at r = b.

The coefficients of the amplitudes A, C, E, and F may be shown in a matrix, Table 1. Equating the determinant value of this matrix to zero yields the dispersion relation for the wave speed.

If the outer boundary is assumed free (Option 2), the fourth boundary condition is replaced by one where the pressure in the fluid is zero for r = b. The resulting matrix of the coefficients is shown in Table 2.

Table 1 - Matrix of coefficients of boundary conditions for impedance tube, Option 1.

Amplitudes →	A	$-iC$	$E/(i\omega)$	$F/(i\omega)$
Continuous normal stress at $r = a$	$(2k^2 - k_s^2)a^2 J_o(q'a)$ $+ 2(q'a)J_1(q'a)$	$2(ka)[J_1(s'a)$ $- (s'a)J_o(s'a)]$	$(\rho_o/\rho_s)(k_s a)^2 J_o(q_o'a)$	$(k_s a)^2(\rho_o/\rho_s)Y_o(q_o'a)$
Zero tangential stress at $r = a$	$2(ka)(q'a)J_1(q'a)$	$(2k^2 - k_s^2)a^2 J_1(s'a)$	0	0
Continuous normal particle vel. at $r = a$	$(q'a)J_1(q'a)$	$(ka)J_1(s'a)$	$-(q_o'a)J_1(q_o'a)$	$-(q_o'a)Y_1(q_o'a)$
Zero normal particle vel. at $r = b$	0	0	$(q_o'b)J_1(q_o'b)$	$(q_o'b)Y_1(q_o'b)$

Table 2 - Matrix of coefficients of boundary conditions for impedance tube, Option 2.

Amplitudes $\rightarrow$	A	$-iC$	$E/(i\omega)$	$F/(i\omega)$
Continuous normal stress at $r = a$	$(2k^2 - k_s{}^2)a^2 J_o(q'a) + 2(q'a)J_1(q'a)$	$2(ka)[J_1(s'a) - (s'a) J_o(s'a)]$	$(k_s a)^2(\rho_o/\rho_s)J_o(q_o'a)$	$(k_s a)^2(\rho_o/\rho_s)Y_o(q_o'a)$
Zero tangential stress at $r = a$	$2(ka)(q'a)J_1(q'a)$	$(2k^2 - k_s{}^2)a^2 J_1^{\cdot}(s'a)$	0	0
Continuous normal particle vel. at $r = a$	$(q'a)J_1(q'a)$	$(ka)J_1(s'a)$	$-(q_o'a)J_1(q_o'a)$	$-(q_o'a)Y_1(q_o'a)$
Zero normal particle vel. at $r = b$	0	0	$J_o(q_o'b)$	$Y_o(q_o'b)$

4.2. Wave Speed in Calibrator

Option 3

For the case of the calibrator, one assumes that the fluid cylinder has a radius a and is bounded by a solid, but not rigid, wall with outer radius b. The following set of boundary conditions applies to the field variables in fluid and cylinder wall.

a. The negative value of the pressure p is equal to the normal stress in the solid at r = a.

b. The tangential stress in the solid is zero at r = a.

c. The normal stress in the solid is zero at r = b.

d. The tangential stress in the solid is zero at r = b.

e. The normal component of the particle velocity in the fluid is equal to that in the solid at r = a.

The resulting coefficient matrix for amplitudes A, B, C, D, and F is shown in Table 3.

It was desirable to analyze the situation where a soft coating was applied to the inside of the calibrator with the purpose of reducing the effective wave speed in the liquid of the calibrator. If this coating is cemented to the wall, the following boundary conditions apply:

Table 3 - Matrix of coefficients of boundary conditions for calibrator, Option 3.

Amplitudes →	A	B	$-iC$	$-iD$	$F/(i\omega)$
Continuous normal stress at $r = a$	$(2k^2 - k_s^2)a^2 J_o(q'a) + 2(q'a)J_1(q'a)$	$(2k^2 - k_s^2)a^2 Y_o(q'a) + 2(q'a)Y_1(q'a)$	$2(ka)[J_1(s'a) - (s'a) J_o(s'a)]$	$2(ka)[Y_1(s'a) - (s'a) Y_o(s'a)]$	$(\rho_o/\rho_s)(k_s a)^2 J_o(q_o'a)$
Zero tangential stress at $r = a$	$2(ka)(q'a)J_1(q'a)$	$2(ka)(q'a)Y_1(q'a)$	$(2k^2 - k_s^2)a^2 J_1(s'a)$	$(2k^2 - k_s^2)a^2 Y_1(s'a)$	0
Zero normal stress at $r = b$	$(2k^2 - k_s^2)b^2 J_o(q'b) + 2(q'b)J_1(q'b)$	$(2k^2 - k_s^2)b^2 Y_o(q'b) + 2(q'b)Y_1(q'b)$	$2(kb)[J_1(s'b) - (s'b) J_o(s'b)]$	$2(kb)[Y_1(s'b) - (s'b) Y_o(s'b)]$	0
Zero tangential stress at $r = b$	$2(kb)(q'b)J_1(q'b)$	$2(kb)(q'b)Y_1(q'b)$	$(2k^2 - k_s^2)b^2 J_1(s'b)$	$(2k^2 - k_s^2)b^2 Y_1(s'b)$	0
Continuous normal particle vel. at $r = a$	$(q'a)J_1(q'a)$	$(q'a)Y_1(q'a)$	$(ka)J_1(s'a)$	$(ka)Y_1(s'a)$	$-(q_o'a)J_1(q'a)$

Option 4

a. The negative value of the pressure p is equal to the normal stress in the solid at $r = a$.

b. The tangential stress in the solid is zero at $r = a$.

c. The normal component of the velocity is zero at $r = b$.

d. The tangential component of the particle velocity is zero at $r = b$.

e. The normal component of the particle velocity is continuous at $r = a$.

The resulting matrix of the coefficients of these boundary conditions is shown in Table 4.

Table 4 - Matrix of coefficients of boundary conditions for coating cemented to calibrator wall, Option 4.

Amplitudes →	A	B	-iC	-iD	F/(iω)
Continuous normal stress at r = a	$(2k^2 - k_s^2)a^2J_o(q'a) + 2(q'a)J_1(q'a)$	$(2k^2 - k_s^2)a^2Y_o(q'a) + 2(q'a)Y_1(q'a)$	$2(ka)\,[J_1(s'a) - (s'a)J_o(s'a)]$	$2(ka)[Y_1(s'a) - (s'a)Y_o(s'a)]$	$(\rho_o/\rho_s)(k_s a)^2J_o(q_o'a)$
Zero tangential stress at r = a	$2(ka)(q'a)J_1(q'a)$	$2(ka)(q'a)Y_1(q'a)$	$(2k^2 - k_s^2)a^2J_1(s'a)$	$(2k^2 - k_s^2)a^2Y_1(s'a)$	0
Zero normal particle vel. at r = b	$(q'b)J_1(q'b)$	$(q'b)Y_1(q'b)$	$(kb)J_1(s'b)$	$(kb)Y_1(s'b)$	0
Zero tangential particle vel. at r = b	$(kb)J_o(q'b)$	$(kb)Y_o(q'b)$	$-(s'b)J_o(s'b)$	$-(s'b)Y_o(s'b)$	0
Continuous normal particle vel. at r = a	$(q'a)J_1(q'a)$	$(q'a)Y_1(q'a)$	$(ka)J_1(s'a)$	$(ka)Y_1(s'a)$	$-(q_o'a)J_1(q_o'a)$

Option 5

If the coating is not cemented to the wall, but is free to move in a tangential direction, condition (d) is replaced by the requirement that the tangential stress in the solid is zero at $r = b$. The resulting matrix is given in Table 5.

Table 5 - Matrix of coefficients of boundary conditions for coating not cemented to calibrator wall, Option 4.

Amplitudes →	A	B	$-iC$	$-iD$	$F/(i\omega)$
Continuous normal stress at r = a	$(2k^2 - k_s^2)a^2J_o(q'a)$ $+ 2(q'a)J_1(q'a)$	$(2k^2 - k_s^2)a^2Y_o(q'a)$ $+ 2(q'a)Y_1(q'a)$	$2(ka)[J_1(s'a)$ $- (s'a)J_o(s'a)]$	$2(ka)[Y_1(s'a)$ $- (s'a)Y_o(s'a)]$	$(\rho_o/\rho_s)(k_sa)^2J_o(q_o'a)$
Zero tangential stress at r = a	$2(ka)(q'a)J_1(q'a)$	$2(ka)(q'a)Y_1(q'a)$	$(2k^2 - k_s^2)a^2J_1(s'a)$	$(2k^2 - k_s^2)a^2Y_1(s'a)$	0
Zero normal particle vel. at r = b	$(q'b)J_1(q'b)$	$(q'b)Y_1(q'b)$	$(kb)J_1(s'b)$	$(kb)Y_1(s'b)$	0
Zero tangential stress at r = b	$2(kb)(q'b)J_1(q'b)$	$2(kb)(q'b)Y_1(q'b)$	$(2k^2 - k_s^2)b^2J_1(s'b)$	$(2k^2 - k_s^2)b^2Y_1(s'b)$	0
Continuous normal particle vel. at r = a	$(q'a)J_1(q'a)$	$(q'a)Y_1(q'a)$	$(ka)J_1(s'a)$	$(ka)Y_1(s'a)$	$-(q_o'a)J_1(q_o'a)$

Empty cylinder

The phase speed in an empty solid cylinder may be readily derived from the matrix in Table 2 by omitting the fifth row and fifth column and setting the determinant value of the remaining 4×4 matrix equal to zero.

Other phase speeds of interest may be obtained. In a fluid column with free boundaries, the phase speed is given by

$$J_o(q'_o a) = 0 \text{ or } (c/c_o)^2 = 1/[1 - j_{o,m}^2/(k_o a)^2] \tag{12}$$

where m = 1,2,... and $j_{o,m}$ are the zeros of the J_0 Bessel function. Thus, there is a cutoff frequency given by

$$\omega_o = j_{o,1} c_o/a \tag{13}$$

below which no wave propagation is possible in a free fluid cylinder. Similarly, the phase speed of waves in a fluid cylinder bounded by rigid walls follows from

$$(q_o a) J_1(q_o a) = 0 \tag{14}$$

or

$$(c/c_o)^2 = 1/[1 - j_{1,m}^2/(k_o a)^2] \tag{15}$$

where m = 0,1,2,... and $j_{1,m}$ are the zeros of the J_1 Bessel function (including $j_{1,0} = 0$).

4.3. Complex Wave Speed in Sample in Impedance Tube

The measurements in the impedance tube consist of a complex reflection coefficient. Thus, it is important to determine the complex wave speed that is connected with a pair of complex elastic moduli. This is accomplished by the program IMPED. Its dispersion relation is the same as in WGUIDE option 1, but the roots are found by a complex root finder described in Ref. 13.

5.0. RESULTS AND DISCUSSION

5.1. Impedance Tube

The original incentive to develop the WGUIDE program was the need to *determine exactly* which propagation speed is measured in the impedance tube. It appears to be tacitly assumed that the dilatational speed is the quantity measured. For instance, in Ref. 14 the sound speed is graphed without further qualification; from the text one may infer that it is the dilatational wave

speed. The present analysis shows that it depends on the circumstances as to what speed applies for the sample in the impedance tube. This is shown in a sequence of figures. The first set shows the dispersion relation in a sample of steel with a density of 7700 kg/m^3, Young's modulus 19.5×10^{10} Pa, shear modulus 8.3×10^{10} Pa, Poisson's ratio 0.37 (data from Ref. 15). The tube is filled with water with density 998 kg/m^3 and wave speed 1481 m/s. The abscissa $k_d a$ is a dimensionless measure of the frequency, with k_d the wave number for the dilatational wave in the solid and a the radius of the cylindrical sample; the ordinate is the ratio of the wave speed to the dilatational wave speed. For large values of $k_d a$ the wave speed approaches the Rayleigh wave speed. Figure 4 shows that for a sample in vacuum at low frequency the bar speed is approached, as expected. With water in the space between the tube wall and the sample, radii a and b, respectively, the wave speed in the sample at low frequency increases with decreasing value of b/a as seen in Figs. 5 and 6 and is indistinguishable from the dilatational wave speed ($c/c_d = 1$) for b/a = 1.001, Fig. 7. Notice that there is a "fluid mode" with low propagation speed in addition to various modes determined by the sample. The same behavior is shown by hard rubber as sample material, density 1100 kg/m^3, shear modulus 0.1×10^{10} Pa, bulk modulus 0.5×10^{10} Pa. Here, however, a thicker layer of fluid is able to constrain the material to manifest dilatational wave speed than for steel (see Figs. 8 through 10). Even for b/a = 1.2, the speed is close to the dilatational wave speed (Fig. 9) The value for b/a is about 1.03 for the NRL-USRD impedance tube.

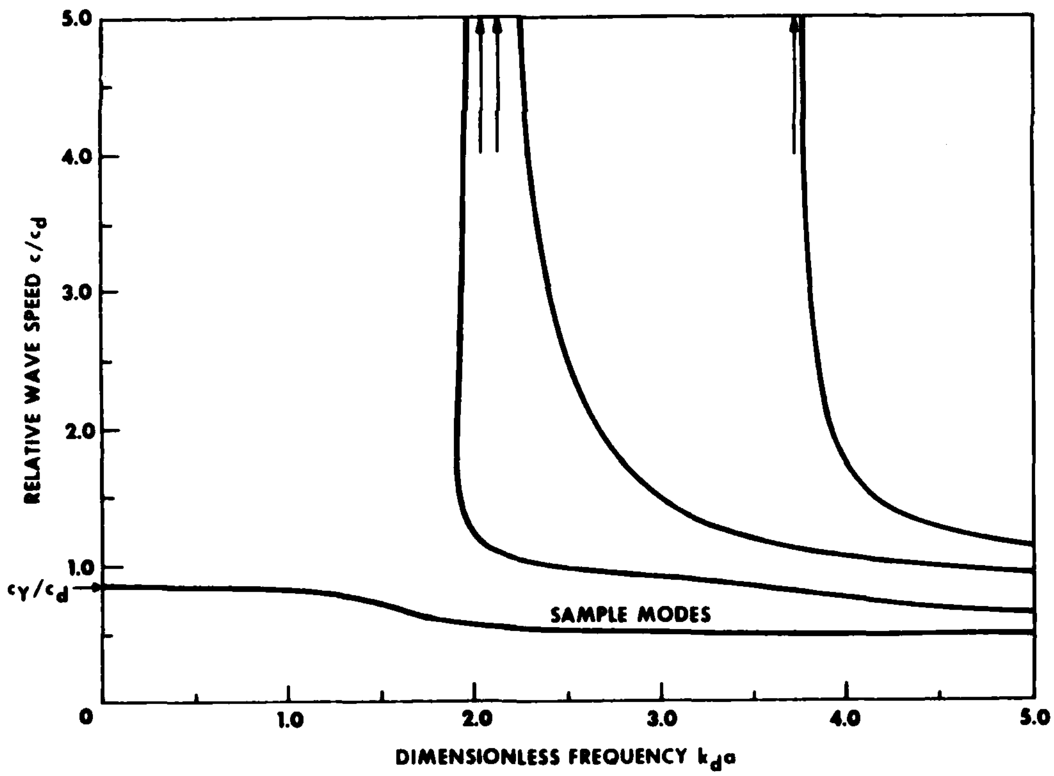

Fig. 4 - Relative wave speed c/c_d as a function of dimensionless frequency $k_d a$ for steel cylinder in vacuum. Vertical arrows indicate asymptotes; c_d is the dilatational speed, c_Y is bar speed.

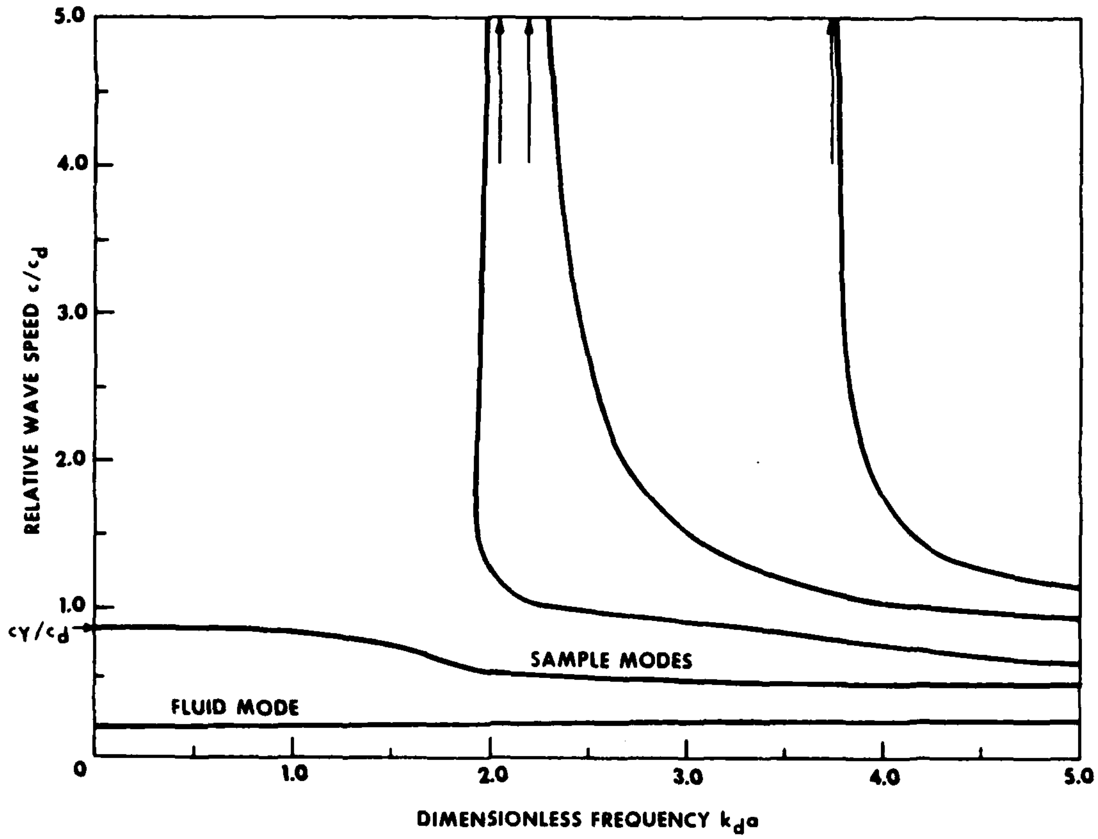

Fig. 5 - Relative wave speed c/c_d as a function of dimensionless frequency k_da for steel cylinder in water, bounded by a concentric rigid wall. Ratio of the cylinder radii is 1.05. Vertical arrows indicate asymptotes; c_d is the dilatational speed, c_Y is the bar speed.

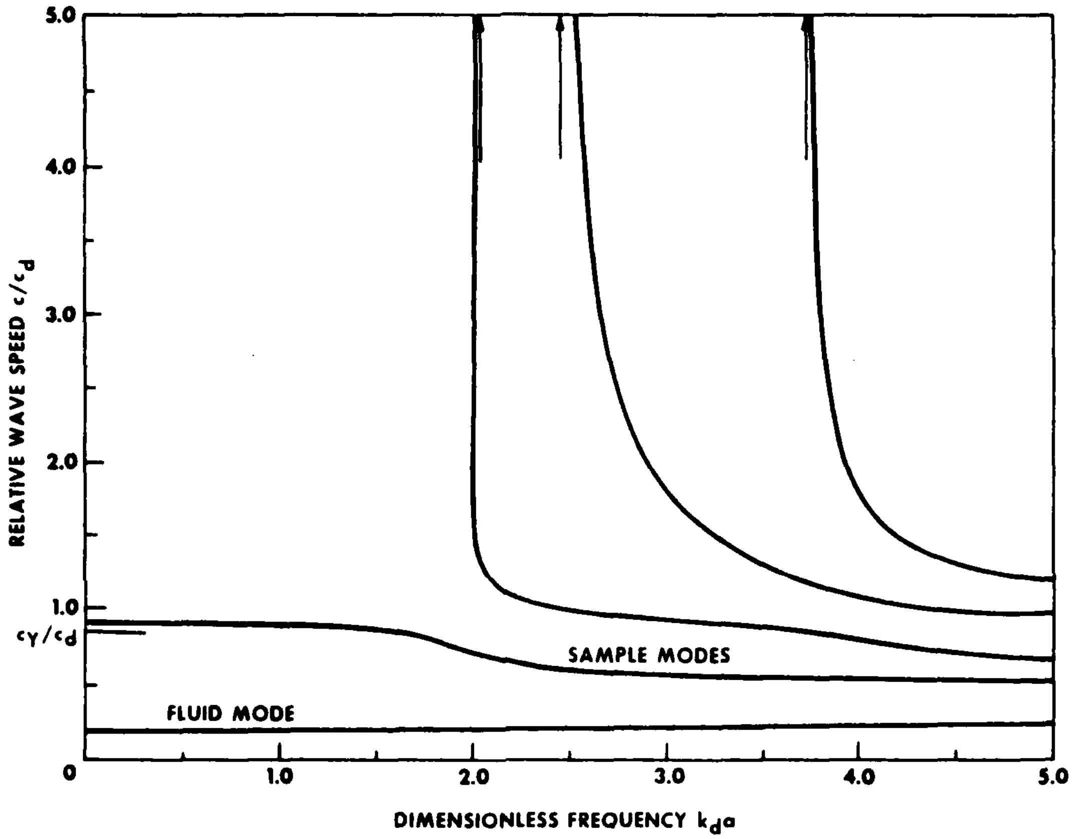

Fig. 6 - Relative wave speed c/c_d as a function of dimensionless frequency k_da for a steel cylinder in water, bounded by a concentric rigid wall. Ratio of the cylinder radii is 1.01. Vertical arrows indicate asymptotes; c_d is the dilatational speed, c_Y is the bar speed.

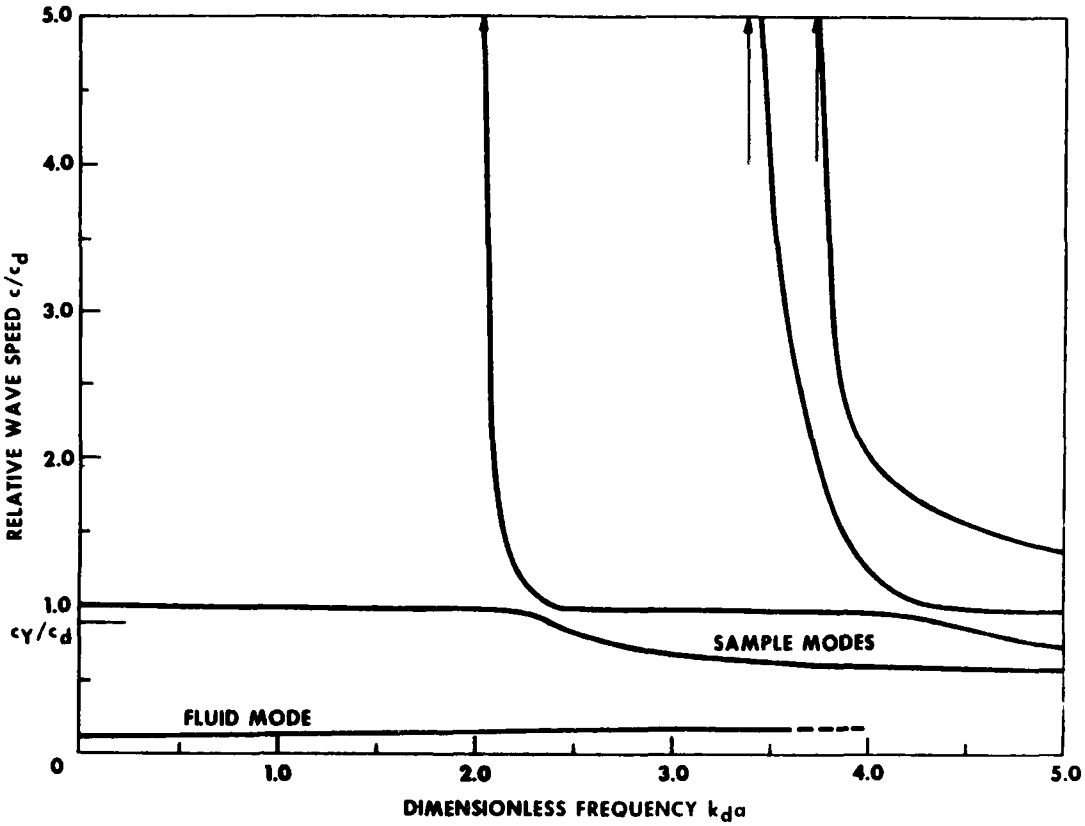

Fig. 7 - Relative wave speed c/c_d as a function of dimensionless frequency $k_d a$ for a steel cylinder in water bounded by a concentric rigid wall. Ratio of cylinder radii is 1.001. Vertical arrows indicate asymptotes; d_d is the dilatational wave speed, c_Y is the bar speed.

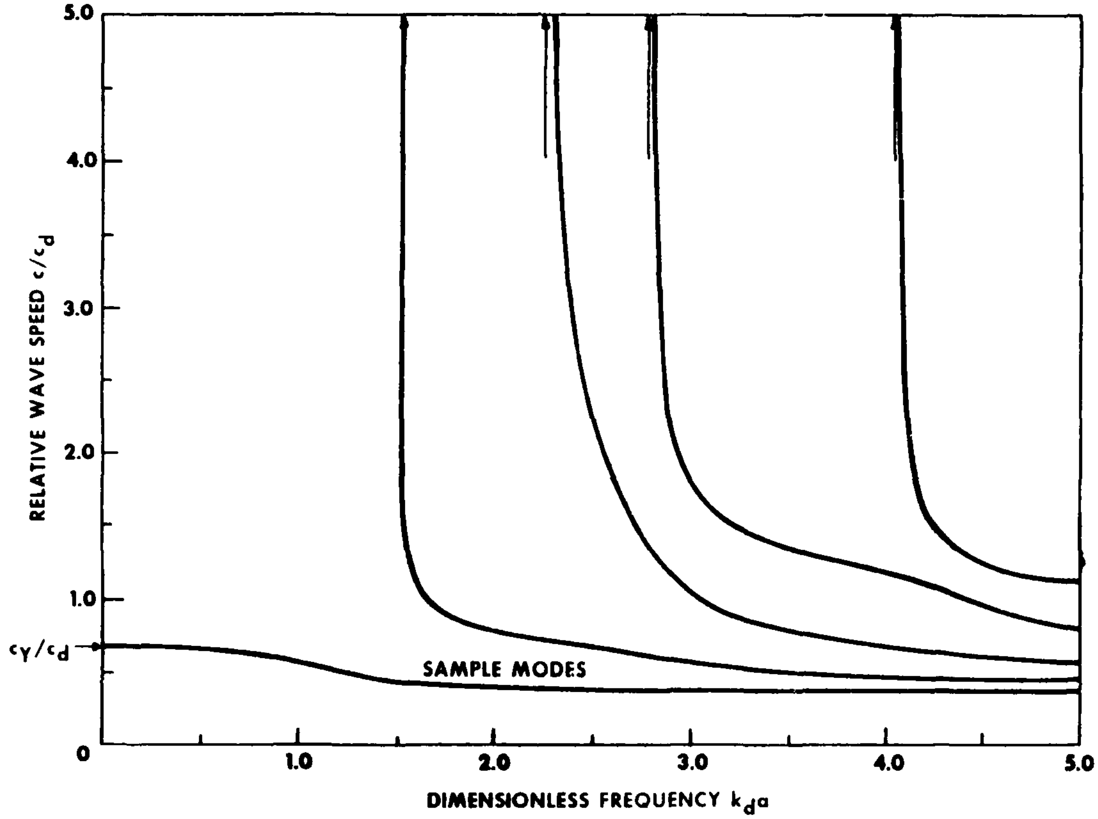

Fig. 8 - Relative wave speed c/c_d as a function of dimensionless frequency $k_d a$ for a rubber cylinder in vacuum. Vertical arrows indicate asymptotes; c_d is the dilatational wave speed, c_Y is the bar speed.

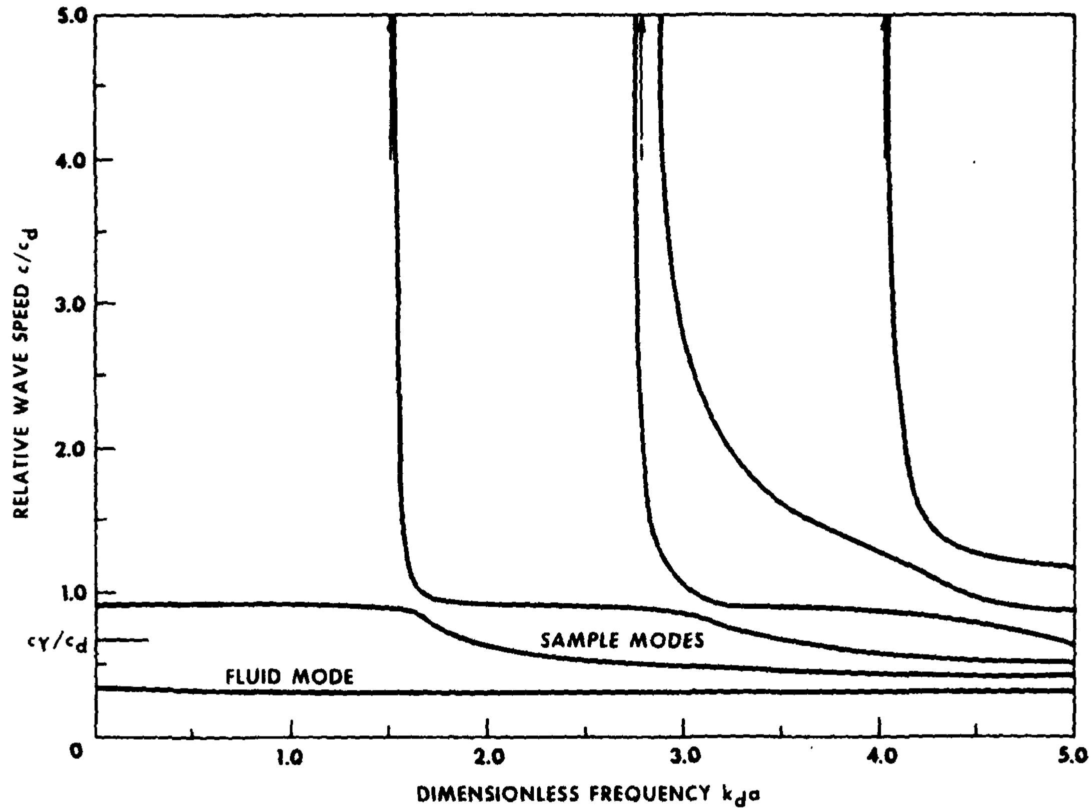

Fig. 9 - Relative wave speed c/c_d as a function of dimensionless frequency $k_d a$ for a rubber cylinder in water, bounded by a concentric rigid wall. Ratio of cylinder radii is 1.2. Vertical arrows indicate asymptotes; c_d is the dilatational wave speed, c_Y is the bar speed.

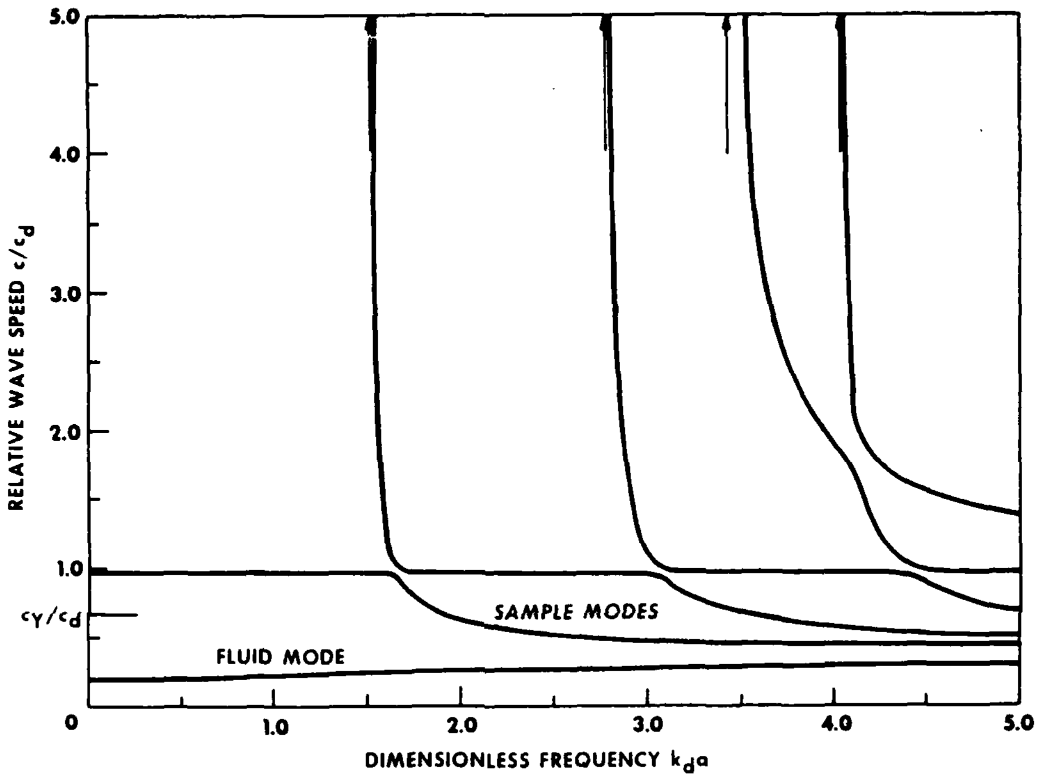

Fig. 10 - Relative wave speed c/c_d as a function of dimensionless frequency k_da for a rubber cylinder in water, bounded by a concentric rigid wall. Ratio of the cylinder radii is 1.05. Vertical arrows indicate asymptotes; c_d is the dilatational wave speed, c_Y is the bar speed.

Generally, for elastomers, the measured wave speed is correctly identified with the dilatational wave speed since the compressibility and density of most rubbers are close to that of water, but this is not true for more rigid materials. A single measurement is not sufficient to determine the pair of elastic moduli needed to describe the material; by preventing the fill fluid to reach the gap between the sample and the tube or by varying the diameter of the sample, one may measure values for different combinations of the moduli from which the desired information may be extracted.

5.2. Calibrator

Two issues of interest were investigated with the WGUIDE program. In the first place, one may consider the influence of the walls on the sound speed in the water. One expects that the less rigid the wall the more the speed will

be reduced. As an example, the dispersion relation was computed for the case of the G40 calibrator developed at NRL-USRD. The following data apply: $a = 9.68$ cm, $b = 10.95$ cm, the material of the wall is aluminum with a density of 2700 kg/m^3, shear modulus 2.4×10^{10} Pa, bulk modulus 7.5×10^{10} Pa and a dilatational wave speed of 6300 m/s (Ref. 15). The results are shown in Fig. 11. It is seen that at low frequency, ($k_d a = 0.1$, which corresponds to 1035 Hz), the ratio of the speed in the tube to that in water is 0.83. Thus, there is a 17% reduction in speed due to the presence of the walls.

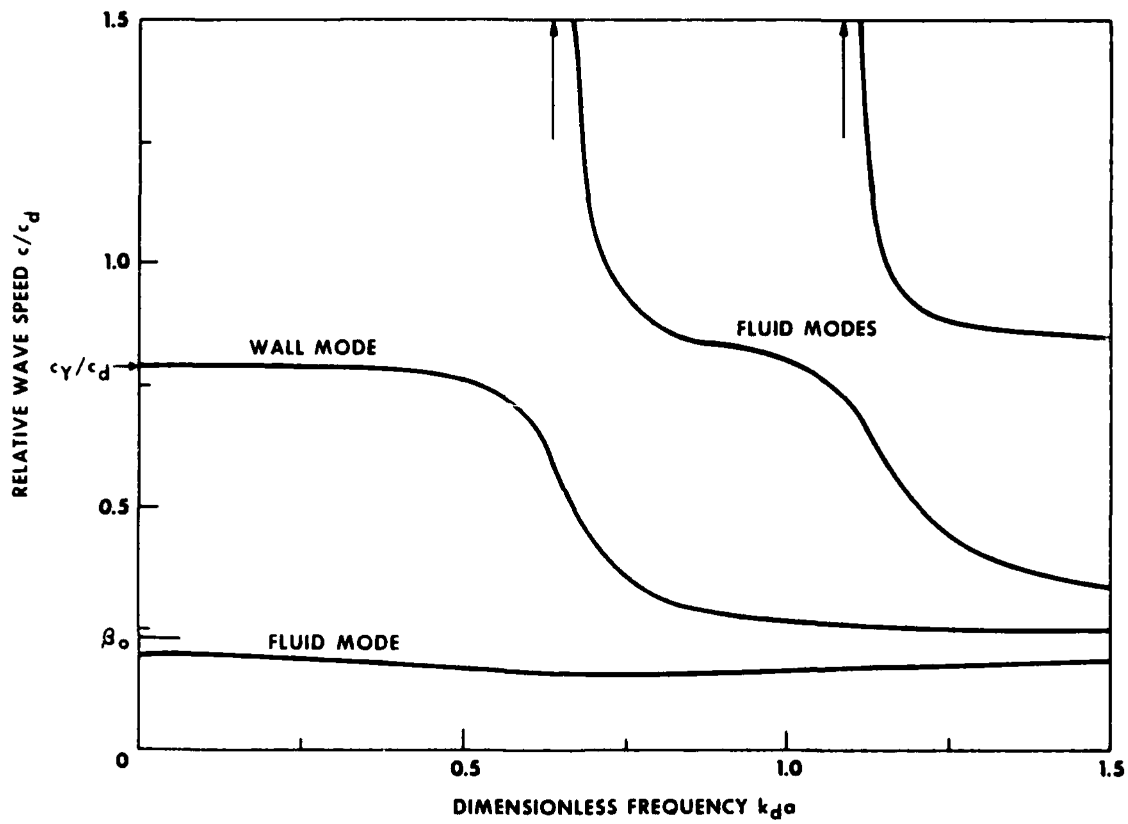

Fig. 11 - Relative wave speed c/c_d as a function of dimensionless frequency $k_d a$ for a water-filled tube of the same cross-sectional dimensions and material as the G40 calibrator. Ratio of the cylinder radii is 1.13. Vertical arrows indicate asymptotes; c_d is the dilatational wave speed, c_Y is the bar speed, c_0 is the compressional wave speed in water.

The next question concerns the calibration of a hot-film particle motion hydrophone in the calibrator. In order to increase the particle velocity for a given pressure amplitude, one can reduce the propagation speed by choosing a less rigid wall material. Rubber might be too weak to support the water column, but a lucite wall might be realizable. Let the dimensions be the same as for the G40. The data for lucite are: density 1200 kg/m^3, shear modulus 0.14×10^{10} Pa, bulk modulus 0.65×10^{10}, dilatational wave speed 2650 m/s. The speed in water for $k_d a = 0.1$ (1035 Hz) is reduced by a factor of 0.30 or a

speed of 440 m/s. Another possible way to reduce the wave speed in the tube is to insert a concentric layer of a suitable material inside the tube. The parameters influencing the reduction in wave speed are the elastic constants, density, and the ratio b/a. The results for a number of materials are shown in Table 6. Only polyethylene gives a sizable reduction in wave speed: a factor of 2. The data for glass, lead, rubber, and lucite were taken from Ref. 15, the data for polyethylene and polystyrene from Ref. 16.

Table 6 - Reduction in wave speed for various wall coatings in G40 calibrator (Frequency is 1035 Hz, a = 7.0 cm, b = 9.8 cm).

MATERIAL	G/K	c_o/c_d	ρ_o/ρ_s	c/c_o
Glass	0.641	0.265	0.434	0.99
Lead	0.131	0.722	0.088	0.86
Lucite	0.215	0.559	0.832	0.84
Polyethylene	0.085	0.760	0.890	0.54
Polystyrene	0.326	0.630	1.065	0.82
Rubber (hard)	0.200	0.617	0.907	0.79

Recent theoretical and experimental work in sound propagation in pipes with acoustic impedance comparable to that in water is found in Refs. 17 and 18.

For comparison with the literature, Fig. 12 shows the dispersion relation in a water-filled tube in addition to the curves for an empty tube, and a water column that has a rigid boundary and one that has a free boundary. The material of the wall is brass, as in Ref. 19, the results of which are reproduced in Ref. 20. The ratio of cylinder radii was chosen accordingly, b/a = 1.13. The material constants for brass are those given in Ref. 15. ρ_s = 8500 kg/m^3, G/K = 0.279, and c_d = 4700 m/s. The "longitudinal wave speed" is given in Ref. 19 as 3590 m/s which actually is closer to the bar speed, 3500 m/s. according to Ref 16. The results in Fig. 12 show that the wave speed approaches the bar speed (subscript Y) for small $k_d a$, in disagreement with Ref. 19 but in accordance with Ref. 21.

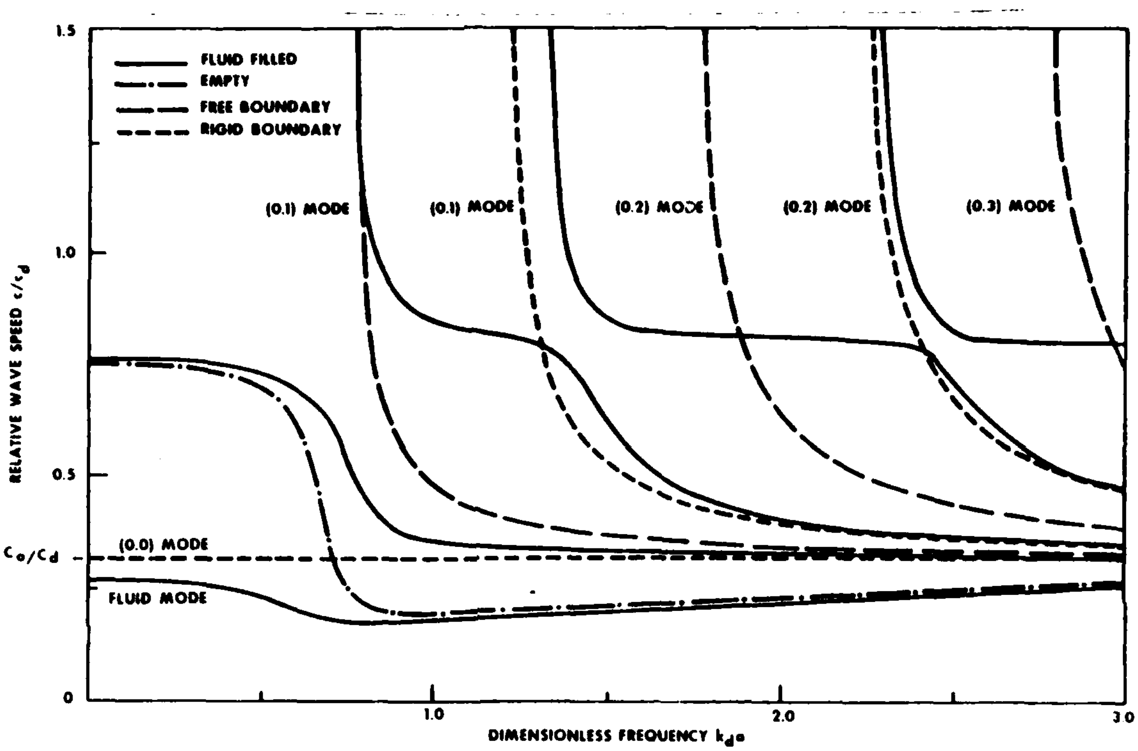

Fig. 12 - Relative wave speed c/c_d as a function of dimensionless frequency $k_d a$ for a water-filled brass tube, according to Ref. 21, with the wave speeds for the separate composing parts: tube and water column. The ratio of the cylinder radii is 1.13; c_d is the dilatational wave speed, c_Y is the bar speed, and c_o is the compressional speed in water. The mode designation for the fluid column follows the nomenclature of Ref. 6 by which the first entry gives the number of radial nodal lines, and the second the number of nodal circles.

6.0. CONCLUSIONS

The mathematical analysis and computer programs developed in this report may be used for determining spatial properties of the acoustic fields, including wave speed, in impedance tube and calibrator. Their application to the interpretation of data from these instruments is subject to the restrictive assumptions underlying the analysis. First, in relating the reflection coefficient to propagation speed in the impedance tube it is assumed that the radial variation of pressure and particle velocity is small or, in other words, that no other than the zero mode of the wave in the sample needs to be considered. In the second place, it is assumed throughout the

computation of wave speeds that the composite cylinders are infinite in the axial direction. This ignores the influence of the unavoidable termination in a finite cylinder on the wave pattern in the waveguide. The experimental efforts to terminate the waveguides by rigid reflectors or other idealized devices can ipso facto be only approximately successful. It is assumed that the idealization implied by the model of infinite cylinders will be more closely approached the larger the ratio of length to diameter of the sections of the waveguide.

Keeping this restriction in mind, one may conclude that it is not necessarily true that the wave speed measured by the impedance tube is the dilatational speed. It is rather a function of the thickness of the fluid-filled gap between sample and wall, in relation to the elastic properties of the sample material that determines the proper wave speed. As a consequence, one may measure different wave speeds by changing the radius of the sample, or by removing the fluid from the gap. Experimental study will have to decide in how far the finite length of the sample would interfere with the interpretation of the data.

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APPENDIX A

Computer Programs

In this appendix the computer programs are described, and listings and examples are presented. An overview of the programs is given in Table A1.

Table A1 - Name and purpose of computer programs*

NAME	TYPE	INPUT	OUTPUT	PURPOSE
RTUBE	Complex	Reflection coefficient	Propagation speed	Impedance tube
WGUIDE Options 1 & 2	Real	G/K and c_o/c_d of sample	Propagation speed tube	Speed in sample in impedance
WGUIDE Options 3-7	Real	G/K and c_o/c_d of wall	Propagation speed	Speed in calibrator
IMPED	Complex	G/K and c_o/c_d of sample	Propagation speed	Speed in sample

* G, shear modulus; K, bulk modulus; c_o speed in water; c_d dilatational speed in solid.

DESCRIPTION OF PROGRAM RTUBE

The computer program RTUBE determines numerical values for the sound speed and attenuation of a sample in the acoustic impedance tube. The input to the program consists of the measured values of the amplitude and phase of the signal reflected by the sample and of the signal reflected without a sample in the tube, indicated by "standard". The program is written in FORTRAN IV and should run on any computer that accepts this language.

There are two major parts in RTUBE. First a real-valued seed or starting value for the sound speed is computed from the reactive part of the acoustic impedance.* This seed is entered into subroutine R0819, which contains the logic for the complex root search. The function, the roots of which are sought, is contained in subroutine DS.**

The real root finder determines the real part of (kd) corresponding to the reactive part X of the impedance Z in Eq. (2). according to the relation

$$\frac{X\ kd}{k_o d} = -(\rho_s/\rho_o)\ \cot(kd)\ . \tag{A1}$$

Obviously, there is an infinite number of roots kd of this equation and one needs circumstantial evidence about the propagation speed in order to choose the correct one. In the iterative procedure used here, it is necessary to evaluate an inverse cotangent function which involves choosing the proper branch.

The propagation speed in the water is computed for a given temperature and pressure. The density of the fluid is 1.034 g/cm^3, applicable to the glycol water mixture used in the experiment. For a different fill-fluid, this number should be changed accordingly.

Important Variables in RTUBE

<u>Input Variables</u>

AMAX Maximum frequency (Hz)

AMIN Minimum frequency (Hz)

DENS ρ_s, density of material (g/cm^3)

PRES p, pressure (MPa)

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* C.M. Ruggiero, "Solution of Transcendental and Algebraic Equations with Application to Wave Propagation in Elastic Plates," NRL Memorandum Report 4449, November 1981.

** P.S. Dubbelday, "Complex Root-Finding Program with Application to the Dispersion Relation of Waves Propagating in a Fluid Plate," NRL Memorandum 4559, November 1981.

PSA	Phase of sample
PST	Phase of standard
STEP	Step size
TEMP	Temperature (°C)
THICK	Length of sample (cm)
VSA	Voltage of sample
VST	Voltage of standard

Computational Variables

A	$(\rho_o/\rho_s)(kd)$
ANS = ANS1	Counter, same as Z plus branch correction ($+n\pi$)
DIFF	Difference of last approximation and current approximation
E	Phase difference (in radians for signals with and without sample).
G	Phase difference corrected for absence of sample
I and X	Slopes computed at seed
J	Tolerance of relative error in root
J1	Number of frequencies
K	Counter
M	Reactance
N	Resistance
Q	2 $k_o d$, k_o is the wave number in water.
U	Number used in slope computation
V	Absolute value of reflection coefficient
W	Wave speed in water corrected for ambient temperature and pressure
Y	cot(kd)

Output variables

AMGA Attenuation (Np/m)

FREQ Frequency (Hz)

SS Sound speed (m/s)

Major Computation Blocks in RTUBE, R0819

Descriptions of the major computation blocks in RTUBE are as follows:

Computation Block	Line Number From	To
Input section	2	25
Slope computation	41	42
Iterative rootfinder	43	59
Output section (real roots)	64	66

Descriptions of the major computation blocks in R0819 are as follows:

Computation Block	Line Number From	To
Set vertical pair	31	32
Test for initial location of vertical pair	39	40
Determination and movement of vertical test pairs	41 52	49 53
Check for iterations	62	63
Check for sign change in vertical pair	67	
Set horizontal pair	68	
Determination and movement of horizontal test pairs	74 86	75 88
Check for sign change in horizontal pair	102	
Check for desired accuracy	111	112
Reduce step size	114	

Horizontal vertical error (real & imaginary)	120	121
Advancing y value	124	
Output section	127	131

```
C
C       RTUBE
C
C       THIS PROGRAM COMPUTES THE SOUND SPEED AND ATTENUATION
C       OF THE IMPEDANCE TUBE WHEN THE VOLTAGE OF THE SAMPLE AND
C       THE VOLTAGE OF THE STANDARD,THE PHASE OF THE SAMPLE AND THE
C       PHASE OF THE STANDARD ARE MEASURED.
C
C       SUBROUTINES USED:
C               R0819
C               DS
C
        REAL M,N,I,J,L,DAT(5,9)
        WRITE(5,10)
10      FORMAT(' ENTER PRESSURE(MPA),LENGTH(CM),TEMP(C),DENSITY(G)   ')
        READ(5,20)PRES,THICK,TEMP,DENS
20      FORMAT(4F10.0)
        THICK=THICK/100
        ZNOT=2.5
        PI=0
        ANS1=-10000
        K=1
        WRITE(5,21)
21      FORMAT(/,' ENTER FREQUENCY (MIN,MAX,STEP) ')
        READ(5,23)AMIN,AMAX,STEP
23      FORMAT(3F10.0)
        OPEN(UNIT=1,NAME='DATXY.DAT',TYPE='NEW',FORM='UNFORMATTED')
        J1=0
        DO 500 FREQ=AMIN,AMAX,STEP
        PI=0
        J1=J1+1
        WRITE(5,25)FREQ
25      FORMAT(//,' FREQUENCY=   ',F10.1,/)
29      WRITE(5,30)
30      FORMAT(' ENTER VOLT. STAND.,VOLT. SAMP, PHASE STAND.,PHASE SAMP')
        READ(5,40,ERR=29) VST,VSA,PST,PSA
40      FORMAT(4F10.0)
        V=VSA/VST
        W=1460.0+10.0*PRES/6.96+2.8*(TEMP-24.0)-.037*(TEMP-24.0)**2.0
        Q=4*3.14159*THICK*FREQ/W
        E=(PSA-PST)/57.2958
        G=E-Q
        IF (ABS(G)-2*3.14159 .LE. 0) GO TO 50
        IF (G .LE. 0)GO TO 60
        G=G-2*3.14159
        GOTO 50
60      G=G+2*3.14159
50      N=(1-V**2)/(1+V**2-2*V*COS(G))
        M=2*V*SIN(G)/(1+V**2-2*V*COS(G))
C       1.0 SHOULD BE CHANGED TO 1.034 FOR GLYCOL-WATER
        A=DENS*2*3.14159*FREQ*THICK/(W*1.0)
        J=.0001
        U=.00001
90      I=-M*(ZNOT-U)+M*(ZNOT+U)
        X=A*(COS(ZNOT-U)/SIN(ZNOT-U))-A*(COS(ZNOT+U)/SIN(ZNOT+U))
        ASSIGN 95 TO JOB
```

```
      IF (ABS(I) .LE. ABS(X))ASSIGN 100 TO JOB
      GO TO JOB
95    Y=A*COS(ZNOT+PI*3.14159 )/SIN(ZNOT+PI*3.14159)
      Z=Y/(-M)
      IF(K .GT. 1000)ASSIGN 100 TO JOB
      GO TO 110
100   H=-M*(ZNOT+PI*3.14159)/A
      Z=ACOS(H/SQRT(1+H**2))
      IF(K .GT. 1000)ASSIGN 95 TO JOB
110   DIFF=ZNOT-Z
      IF(ABS(DIFF) .LE. J) GO TO 200
      ZNOT=Z
      K=K+1
      IF(K .GT. 1001)ZNOT=ANS1
      IF (K .GT. 1001)K=1
      GOTO JOB
200   ANS=Z+PI*3.14159
      IF(ANS1 .GT. (ANS ))PI=1
      IF(ANS1 .GT. ANS)ZNOT=ANS1
      IF(ANS1 .GT. (ANS))GOTO JOB
      WRITE(5,210)ANS,M,N
210   FORMAT(' REAL SEED, REACTANCE=, RESISTANCE=  ',3F10.5)
      WRITE(1)FREQ,M,N,A,ANS
      ANS1=ANS
500   CONTINUE
      CLOSE(UNIT=1)
      CALL R0819(J1,THICK)
      END
```

```
      SUBROUTINE R0819(J1,THICK)
C
C     THE PURPOSE OF THIS SUBROUTINE IS TO DETERMINE THE
C     COMPLEX ROOTS OF FUNCTION DS
C
C     DESCRIPTION OF PARAMETERS
C             J1 -THE NUMBER OF FREQUENCIES TO BE COMPUTED
C             THICK -THE THICKNESS OF THE ELEMENTS
C
C
C     FUNCTION REQUIRED
C             Z - APPROXIMATE ROOT
C
C     USAGE
C             DS(Z)
C
C     NOTE:
C             ALL DATA IS PASSED IN A COMMON STATEMENT
      COMPLEX A11,A22,S11,S22,DELA,DELS,Z,F,DSP,AI,DFDZ,DZ,ZVAR,
     1Z1,Z2,Z3,Z4,F1,F2,F3,F4,CSTH,FF,DSP2,AKAP,ACOTZ,COTZ,SINZ,COSZ
      COMPLEX DS,CSQRT,CMPLX,CCOS,CSIN,CHECK,TEST
      REAL LOOK
      COMMON RHO,AKSD,A
      OPEN(UNIT=1,NAME='DATXY.DAT',TYPE='OLD',FORM='UNFORMATTED')
      K=1
      ANR=8
      TOL=.01
      REDF=10
      DELZ=.01
      KAM=10000
      DO 10 I=1,J1
      READ(1)FREQ,AKSD,RHM,A,ZNOT

      ZNOT=ZNOT/THICK
      RHO=0
      AI=CMPLX(0.,1.)
      NR=INT(ANR)
      NRV=0
C     COMPUTE STEPSIZE DELZ
      ZVAR=ZNOT
      Z=ZVAR+DELZ
      DFDZ=DS(Z)/DELZ
      RHO=RHO+RHM/ANR
      NRV=NRV+1
      DZ=-DS(ZVAR)/DFDZ
      X=ABS(REAL(DZ))
      Y=ABS(AIMAG(DZ))
      DEL=AMAX1(X,Y)*1
      DELP=DEL
   70 DEL=DELP
   64 Z1=ZVAR-AI*DEL
      Z2=ZVAR+AI*DEL
   65 F1=DS(Z1)
      F2=DS(Z2)
      KR=0
      KL=0
```

```
      KUP=0
      KDOWN=0
      SIGN=AIMAG(F1)*REAL(F2)-REAL(F1)*AIMAG(F2)
      IF(SIGN)71,72,73
71    FH1=-1.
      GO TO 74
72    WRITE(5,302) SIGN
302   FORMAT (' LEFT-RIGHT SIGN = ',E12.5)
      GO TO 92
73    FH1=1.
74    DELH=FH1*DEL
75    Z1=Z1+DELH
      Z2=Z2+DELH
      F1=DS(Z1)
      F2=DS(Z2)
      SIGN=AIMAG(F1)*REAL(F2)-AIMAG(F2)*REAL(F1)
      IF (SIGN)  81,82,83
81    FH2=-1.
      KL=KL+1
      IF (KAM-KL) 99,99,84
99    WRITE(5,310)
310   FORMAT (' EXIT LEFT')
      GO TO 100

82    GO TO 92
83    FH2 = 1.
      KR=KR+1
      IF(KAM-KR) 98,98,84
98    WRITE (5,311)
311   FORMAT(' EXIT RIGHT')
      GO TO 100
84    IF (FH1*FH2)  92,75,75
92    Z4=(Z1+Z2)/2.+DEL
D     WRITE (5,886) KR,KL
D886  FORMAT ('   KR=',I4,'   KL=',I4)
      KR=0
      KL=0
      Z3=(Z1+Z2)/2.-DEL
      F3=DS(Z3)
      F4=DS(Z4)
      SIGN=AIMAG(F3)*REAL(F4)-AIMAG(F4)*REAL(F3)
      IF (SIGN) 101,102,103
101   FV1=1.
      GO TO 104
102   WRITE (5,304) SIGN
      GO TO 122
103   FV1=-1.
104   DELV=FV1*DEL
105   Z4=Z4+AI*DELV
      Z3=Z3+AI*DELV
      F3=DS(Z3)
      F4=DS(Z4)
      SIGN=AIMAG(F3)*REAL(F4)-AIMAG(F4)*REAL(F3)
304   FORMAT(' UP-DOWN SIGN =',E12.5)
      IF (SIGN) 111,112,113
111   FV2 =1.
```

```
      KUP=KUP+1
      IF(KAM-KUP) 97,97,114
97    WRITE(5,305)
305   FORMAT (' EXIT UP')
      GO TO 100
112   GO TO 122
113   FV2=-1.
      KDOWN=KDOWN+1
      IF (KAM-KDOWN) 96,96,114
96    WRITE(5,306)
306   FORMAT(' EXIT DOWN')
      GO TO 100
114   IF (FV1*FV2) 122,105,105
122   REGA=REAL(Z1)
D     WRITE(5,888) KUP,KDOWN
D888  FORMAT ('    KUP=',I4,'    KDOWN=',I4)
      AMGA=AIMAG(Z3)
      KUP=0
      KDOWN=0
      IF (AMGA) 133,134,133
134   WRITE (5,233)
233   FORMAT(' AMGA IS ZERO')
      GO TO 131
133   AMGA=ABS(AMGA)
      IF(DEL/AMGA-TOL)132,132,131
131   CONTINUE
      DEL=DEL/REDF
      Z1=(Z3+Z4)/2.-AI*DEL
      Z2=(Z3+Z4)/2.+AI*DEL
      GO TO 65
132   REGA=REAL (Z1) -DELH/2.
      AMGA=AIMAG(Z3)-DELV/2.
      DELH=ABS(DELH/2.)
      DELV=ABS(DELV/2.)
      IF(NR-NRV)145,145,146
146   ZVAR=REGA+AMGA*AI
      RHO=RHO+RHM/ANR
      NRV=NRV+1
      GO TO 70
145   WRITE (5,173)FREQ
173   FORMAT(/,' SOUND SPEED AND ATTENUATION FOR FREQ =:  ',F6.0)
      SS=(2*3.14159*FREQ)/REGA
      WRITE (5,174)SS,ABS(AMGA)
174   FORMAT(2E15.5)
10    CONTINUE
      CLOSE(UNIT=1)
100   END
```

```
      FUNCTION DS(Z)
C
C     THIS IS A SPECIFIC FUNCTION  TO BE CALCULATED WITH
C     COMPLEX ROOTFINDER.  THE COMPLEX ARGUMENT IS Z.
C
C     NOTE
C             ALL DATA IS PASSED IN COMMON STATEMENT
      COMPLEX DS,Z,A11,A12,A22,ACOTZ,COTZ,SINZ,COSZ,CCOS,CSIN,S11,CMPLX
      COMMON RHO,AKSD,THICK,A
13    FORMAT(4F10.5,2E15.5)
      AI=CMPLX(0.0,1.0)
      A12= -THICK*Z
      A11=CMPLX(-AKSD,RHO)
      A22=A12*A11
      S11=THICK*Z
      COSZ=CCOS(S11)
      SINZ=CSIN(S11)
      COTZ=COSZ/SINZ
      ACOTZ=A*COTZ
      DS=ACOTZ+A22
      RETURN
      END
```

EXAMPLE:

```
RUN RTUBE <CR>
ENTER PRESSURE(MPA),LENGTH(CM),TEMP(C),DENSITY(G)
.61,12.09,23.9,1.13 <CR>

ENTER FREQUENCY (MIN,MAX,STEP)
4000,11000,1000  <CR>


FREQUENCY=      4000.0

ENTER VOLT. STAND.,VOLT. SAMP, PHASE STAND.,PHASE SAMP
.084,.077,269.91,287.41  <CR>
REAL SEED, REACTANCE=, RESISTANCE=     2.22014   0.77738   0.06983


FREQUENCY=      5000.0

ENTER VOLT. STAND.,VOLT. SAMP, PHASE STAND.,PHASE SAMP
.127,.112,216.6,237.94  <CR>
REAL SEED, REACTANCE=, RESISTANCE=     2.76703   2.61257   0.50517


FREQUENCY=      6000.0

ENTER VOLT. STAND.,VOLT. SAMP, PHASE STAND.,PHASE SAMP
.183,.148,176.49,192.52 <CR>
REAL SEED, REACTANCE=, RESISTANCE=     3.35705  -4.64150   4.21691


FREQUENCY=      7000.0

ENTER VOLT. STAND.,VOLT. SAMP, PHASE STAND.,PHASE SAMP
.231,.173,136.86,145.43  <CR>
REAL SEED, REACTANCE=, RESISTANCE=     3.77241  -1.44402   0.46431


FREQUENCY=      8000.0

ENTER VOLT. STAND.,VOLT. SAMP, PHASE STAND.,PHASE SAMP
.263,.177,101.86,111.03 <CR>
REAL SEED, REACTANCE=, RESISTANCE=     4.28325  -0.48575   0.24388


FREQUENCY=      9000.0

ENTER VOLT. STAND.,VOLT. SAMP, PHASE STAND.,PHASE SAMP
.308,.190,71.89,85.73  <CR>
REAL SEED, REACTANCE=, RESISTANCE=     4.79123   0.08435   0.23873


FREQUENCY=     10000.0

ENTER VOLT. STAND.,VOLT. SAMP, PHASE STAND.,PHASE SAMP
.338,.220,40.17,53.37  <CR>
REAL SEED, REACTANCE=, RESISTANCE=     5.24966   0.64499   0.30552


FREQUENCY=     11000.0

ENTER VOLT. STAND.,VOLT. SAMP, PHASE STAND.,PHASE SAMP
.339,.242,358.36,9.79  <CR>
REAL SEED, REACTANCE=, RESISTANCE=     5.72622   1.75329   0.75410
```

```
SOUND SPEED AND ATTENUATION FOR FREQ =:    4000.
   0.13677E+04     0.45741E+00

SOUND SPEED AND ATTENUATION FOR FREQ =:    5000.
   0.13658E+04     0.60050E+00

SOUND SPEED AND ATTENUATION FOR FREQ =:    6000.
   0.13943E+04     0.88003E+00

SOUND SPEED AND ATTENUATION FOR FREQ =:    7000.
   0.14196E+04     0.10834E+01

SOUND SPEED AND ATTENUATION FOR FREQ =:    8000.
   0.14255E+04     0.14734E+01

SOUND SPEED AND ATTENUATION FOR FREQ =:    9000.
   0.14290E+04     0.19029E+01

SOUND SPEED AND ATTENUATION FOR FREQ =:   10000.
   0.14401E+04     0.19019E+01

SOUND SPEED AND ATTENUATION FOR FREQ =:   11000.
   0.14429E+04     0.15798E+01
```

1) **All underlined portions are user supplied.**

2) <CR> indicates 'RETURN'

DESCRIPTION OF PROGRAM WGUIDE

The program WGUIDE computes the speed of axially symmetric waves propagating along the axial direction in composite infinite cylindrical waveguides, given the values of the ratio of shear modulus and bulk modulus of the solid and the ratio of the propagation speed in the fluid to the dilatational wave speed in the solid. To facilitate tracing the complete graph for the dispersion relation, including higher modes, two methods of scanning are possible. In the first one, the dimensionless frequency $k_d a$ (abscissa) is fixed and the program searches for all relative wave speeds (ordinate) between two limits. In the other method, the program steps through increasing or decreasing values for the dimensionless frequency and finds the smallest value for the relative wave speed that falls between limits, chosen such that the desired branch is bracketed. It is also possible to determine the vertical asymptotes. The application of the program is twofold: firstly, the program computes the wave speed in the sample inside an impedance tube which is assumed to be rigid and separated from the tube wall by a concentric layer of fluid; secondly, one may apply the program to assess the influence of nonrigidity of the wall for inertial calibrators.

The main program calls subroutine FUNC for Option numbers 1 through 5. FUNC, in turn, calls subroutine DI3 which contains the impedance tube dispersion relation, for Options 1 and 2. Number 1 assumes that the impedance tube wall is rigid, number 2 (rather unrealistically) assumes a free outer boundary. For Option numbers 3, 4, and 5, FUNC calls subroutine CALIB, which contains the dispersion relations for the inertial calibrator arrangement. Number 3 is for an elastic tube, fluid-filled or empty; number 4 is for a rigid tube with inner coating cemented to the wall; and number 5 is for a rigid tube with inner coating free to move tangentially with respect to the wall. If the Option number is 6 or 7, the main program calls subroutine FLUID; for number 6, it computes the wave speed for a fluid tube with rigid boundary. Note that the zero order is the compressional wave speed in the fluid. For the number 7, the wave speed in a fluid with free boundary (in this case there is a low-frequency cut-off) is calculated.

The program computes the location of vertical asymptotes by entering a negative value of AKDA. In Option 1, entering a negative value fo RHM will prompt the computation of the wave speed in a solid cylinder in vacuum (indicated by "VACUUM TUBE" in the listing and printout). In Option 3, setting RHM equal to 0 will result in the computation of the wave speed in an empty cylindrical shell, indicated by "EMPTY TUBE" in print-out.

The Bessel function subroutines are documented in: System/360 Scientific subroutine Package Version II (c), International Business Machines Corporation 1966, 1967, 1968, White Plains, NY, Section - Special Operations and Functions, 363 - 367 (copies of these subroutines will not be found in this report; but the user may implement the needed Bessel function routines from the program library). Subroutine DET computes the determinant value of a matrix. The various results are stored by calling subroutine FILE, which is prompted by choosing Option number 9. The values of $\beta_Y = c_Y/c_d$, $\beta_s = c_s/c_d$, $\beta_R = c_R/c_d$, $\beta_o = c_o/c_d$, and the asymptotes are also stored in a

file separate from the dispersion results in order to discriminate between them. They can, however, appear on the same graph by running a plotting program named PLOTTER.*

FORTRAN Variable Names for WGUIDE

Input Variables

KB: Option number determining mode of operation
- = 1 Sample in fluid tube with rigid boundary
- = 2 Sample in fluid tube with free boundary
- = 3 Fluid-filled elastic tube or empty tube
- = 4 Fluid-filled coated rigid tube (cemented)
- = 5 Fluid-filled coated rigid tube (not cemented)
- = 6 Fluid tube with rigid boundary
- = 7 Fluid tube with free boundary

GOK (G/K) ratio of shear modulus to bulk modulus

COCD (c_o/c_d) ratio of wave speed in fluid to dilatational wave speed in solid

RHM (ρ_o/ρ_s) ratio of fluid density to density of solid; negative value for solid cylinder in vacuum; zero for empty calibrator

AKDA ($k_d a$) dimensionless wave number of dilatational waves

$$k_d a = \frac{\omega a}{c_d}$$

B1 or BAM Minimum ($k_d a$ if asymptote option, β otherwise)

B2 Maximum ($k_d a$ if asymptote option, β otherwise)

DB or DEL Step size of ($k_d a$ if asymptote option, β otherwise)

KO: Return option number
- = 1 program returns to entering input option number
- = 2 program returns to entering GOK
- = 3 program returns to entering COCD
- = 4 program returns to entering RHM
- = 5 program returns to entering MIN, MAX, STEP (Y value)
- = 6 program returns to entering BDA
- = 7 program returns to entering AKDA
- = 8 program returns to entering MIN, MAX, STEP (X value)
- = 9 stores data
- 10 program exits

* C.M. Ruggiero and R.W. Anderson, "General Purpose High-Resolution Plotting Package for Tektronix 4662 Plotter and Compatible CRT Terminals," NRL Memorandum Report 4533, August 1981.

AKM	Maximum $k_d a$
DKDA	Step size of $k_d a$

Computational Variables

A	Array that stores $k_d a$
B	Array that stores β
D1 and D2,	Values of the determinant
TOL	Tolerance

Output Variables

AKF	Asymptote at $k_d a$
ERR	Error (DEL/2)
AKDA	$k_d a$
BAF	$\beta = c/c_d$, final value

Major Computation Blocks in WGUIDE

Descriptions of the major computation blocks in WGUIDE are as follows:

Computation Block	Line Number From	To
Input Section	4	55
Computation of Asymptotes	61	78
Bisection Algorithm	89	111
Output Section	112	113

SOURCE LANGUAGE LISTING

```
C
 C      WGUIDE
C       WRITTEN BY P. DUBBELDAY AND T. RUGGIERO
C
C       THE PURPOSE OF THIS PROGRAM IS TO CALCULATE THE SPEED
C       OF AXIAL WAVES IN CYLINDRICAL WAVE GUIDES.
C
C       C       SUBROUTINES: BESJ,BESY,BESK,DET,FILE,FUNC,INUE
C                       IO,DI3,CALIB,FLUID
C
        REAL A(500),B(500)
        COMMON AKA,AKDA,BAMS,ACC,COCD,BDA,RHM,KB,AA
        K9=0
110     WRITE(5,120)
120     FORMAT(' ENTER # CORRESPONDING TO OPTION DESIRED',
        1/,'(1) SAMPLE IN FLUID TUBE WITH RIGID BOUNDARY',
        2/,'(2) SAMPLE IN FLUID TUBE WITH FREE BOUNDARY',
        3/,'(3) FLUID FILLED ELASTIC TUBE, OR EMPTY TUBE',
        4/,'(4) FLUID FILLED COATED RIGID TUBE (CEMENTED)',
        5/,'(5) FLUID FILLED COATED RIGID TUBE (NOT CEMENTED)',
        6/,'(6) FLUID TUBE WITH RIGID BOUNDARY',
        7/,'(7) FLUID TUBE WITH FREE BOUNDARY')
        READ (5,640) KB
        IF(KB .GE. 6)CALL FLUID(KB)
        TOL=.001
        ACC=.01
130     WRITE (5,140)
140     FORMAT(/,'$ENTER RATIO OF SHEAR TO BULK MODULUS (G/K)  ')
        READ (5,170) GOK
        IF(K9 .EQ. 1)GO TO 300
150     WRITE (5,160)
160     FORMAT ('$ENTER RATIO OF FLUID TO SOLID SPEED (CO/CD)  ')
        READ(5,170) COCD
170     FORMAT (F15.0)
        IF(K9 .EQ. 1)GO TO 300
180     WRITE(5,190)
190     FORMAT(/,' ENTER RATIO OF FLUID TO SOLID DENSITY ',
        1/,'$ (FOR EMPTY TUBE SET RHM=0,VACUUM TUBE SET RHM=NEG VALUE)
        READ(5,170)RHM
        IF(K9 .EQ. 1)GO TO 300
        BAMS=SQRT(GOK/(1.+4.*GOK/3.))
        K0=7
        BY=BAMS*3./SQRT(3.+GOK)
        POI=(3.-2.*GOK)/(6.+2.*GOK)
        BRE=(0.87+1.12*POI)*BAMS/(1.+POI)
        WRITE(5,200)BAMS,BY,BRE
200     FORMAT(/,' SHEAR= ',E8.3,' YOUNG= ',E8.3,' RAYLEIGH= ',E8.3)
C
C       IN THE FIRST 3 STORAGE LOCATIONS ARE POINTS OF THE YOUNGS
C       MODULUS.  IN THE 4TH LOCATION CO/CD IS STORED AT .1
        A(1)=0.5
        B(1)=COCD
        A(2)=1.
        B(2)=COCD
        A(3)=2.
        B(3)=COCD
```

```
      A(4)=.1
      B(4)=BY
C
C     INPUT SECTION
C
      N=4
  210 WRITE(5,290)
      READ(5,260)AKDA
      IF(K9.EQ. 1)GOTO 300
      IF(AKDA .GE. 0)GO TO 230
      WRITE(5,220)
  220 FORMAT(/,'$ENTER MIN,MAX,STEP  (HORIZONTAL SCAN)  ')
      GO TO 250
  230 WRITE (5,240)
  240 FORMAT (/,'$ENTER MIN,MAX,STEP  (VERTICAL SCAN)  ')
  250 READ (5,260) B1,B2,DB
  260 FORMAT(3E15.0)
      IF(K9 .EQ. 1)GOTO 300
  270 WRITE (5,280)
  280 FORMAT (/,'$ENTER RATIO OF TUBE TO SAMPLE RADIUS (B/A)  ')
      READ (5,170) BDA
      TYPE *, ' '
  290 FORMAT(/,'$ENTER X VALUE  (SET KDA NEGATIVE FOR ASYMPTOTES)  ')
  300 IF (AKDA) 310,440,440
  310 KO=8
      AKDA=B1
  320 DEL =DB
      N=N+1
C
C     FUNC IS A SUBROUTINE THAT CALLS THE CORRECT SUBROUTINE TO
C        EVALUATE THE DETERMINANT. KB IS THE OPTION DESIRED.
C
      CALL FUNC(-1,KB,D1)
  330 AKDA=AKDA +DEL
      CALL DECLAR
  340 CALL FUNC(-1,KB,D2)
  350 IF(D1*D2) 370,540,390
  360 GO TO 620
  370 IF (ABS(DEL/AKDA)-TOL) 420,380,380
  380 AKDA=AKDA-DEL
      DEL=DEL/10.
      GO TO 330
C
C     CHECKS IF K VALUE EXCEEDS MAXIMUM CHOSEN
C
  390 IF (AKDA-B2) 330,400,400
  400 WRITE(5,410)
  410 FORMAT (' NO MORE ASYMPTOTES ')
      GO TO 620
  420 ERR=DEL/2.
      AKF=AKDA-ERR
      WRITE (5,430) AKF,ERR
  430 FORMAT (' ASYMPTOTE AT X = ',F10.5,' +/- ',E9.3)
C
C     THE ASYMPTOTE ARE FOUND AND POINTS STORED AT Y=1.25,1.5
C
```

```
      A(N) = AKF
      B(N)=1.25
      N=N+1
      A(N)=AKF
      B(N)=1.5
      GO TO 320
440   BAM=B1
450   DEL=DB
      N=N+1
      CALL FUNC(BAM,KB,D1)
460   BAM=BAM+DEL
      CALL DECLAR
470   CALL FUNC(BAM,KB,D2)
480   IF (D1*D2) 490,540,510
490   IF (ABS(DEL/BAM)-TOL) 560,500,500
500   BAM=BAM-DEL
      DEL=DEL/10.
      GO TO 460
510   IF (BAM-B2) 460,520,520
520   WRITE (5,530)
530   FORMAT (' NO MORE ROOTS')
      GO TO 620
C
C     PRINTED WHEN EITHER DETERMINANT IS EQUALTO ZERO
540   WRITE (5,550)
550   FORMAT (' D1*D2=ZERO')
      GO TO 620
560   ERR=DEL/2.
      BAF=BAM-ERR
570   IF(KO .NE. 7)GO TO 600
580   WRITE (5,590) BAF,ERR
590   FORMAT(' Y =  ',F10.5,'  ERROR = ',E9.2 )
      A(N)=AKDA
      B(N)=BAF
      GO TO 450
600   WRITE (5,610) AKDA,BAF,ERR
610   FORMAT(' X = ',F10.5,' Y = ',F10.5,' ERROR = ',E9.3)
      A(N)=AKDA
      B(N)=BAF
      AKDA=AKDA+DKDA
      IF ((AKM-AKDA)*DKDA) 620,440,440
620   K9=1
C     TO RECYCLE BACK OR EXIT CHOSE OPTION #
      WRITE(5,630)
630   FORMAT(/,' ENTER # CORRESPONDING TO CHANGE '
      1/,' 1) OPTION'
      2/,' 2) SHEAR MODULUS/BULK MODULUS (G/K)   '
      3/,' 3) SPEED IN FLUID/SPEED IN SOLID (CO/CD)'
      4/,' 4) FLUID DENSITY/SOLID DENSITY (RHM)   '
      5/,' 5) MIN MAX, STEP (Y VALUES)'
      6/,' 6) TUBE RADIUS/SAMPLE RADIUS (B/A)   '
      7/,' 7) X VALUE (KDA)'
      8/,' 8) MIN,MAX,STEP (X VALUES)'
      9/,' 9) STORE DATA'
      9/,' 10) EXIT FROM PROGRAM')
      N=N-1

      READ (5,640) KO
640   FORMAT (I4)
      GO TO (110,130,150,180,230,270,210,660,650,690) KO
650   CALL FILE(N,A,B)
      GO TO 620
660   WRITE (5,670)
670   FORMAT(/,'$ENTER X VALUES: START(KNOWN),END,STEP(+/-)   ')
      READ (5,680) AKDA,AKM,DKDA
680   FORMAT (3F15.0)
      GO TO 230
690   CONTINUE
      END
```

SUBROUTINES

SUBROUTINE: DET

```
C
C         SUBROUTINE DET
C         DECEMBER, 1980
C         EDITED BY TINA RUGGIERO
C
          SUBROUTINE DET(N,A,D)
C
C         THIS SUBROUTINE USES THE METHOD OF GAUSSIAN ELIMINATION
C         TO CALCULATE THE DETERMINANT
C
          DOUBLE PRECISION A(5,5),B(5),X(5),D,SUM,C
          DIMENSION JPRM(5)
          D=1.0D 00
          DO 13 I=1,N
          X(I)=0.0D 00
 13       JPRM(I)=I
C
C         FIND THE ELEMENT OF MAXIMUN ABSOLUTE VALUE
C
          DO 1 K=1,N
          C=A(K,K)
          II=K
          JJ=K
          DO 2 J=K,N
          DO 2 I=K,N
          IF (DABS(C)-DABS(A(I,J)))3,2,2
  3       C=A(I,J)
          II=I
          JJ=J
  2       CONTINUE
          D=D*C
          IF(DABS(D))20,20,30
 20       WRITE(5,100)
100       FORMAT(' MATRIX A(N,N)IS SINGULAR')
          D=0.0
          RETURN
 30       B(II)=B(II)/C
          KPO=K+1
C
C         DIVIDE EACH ELEMENT OF THE II TH ROW BY C
C
          DO 4 J=K,N
  4       A(II,J)=A(II,J)/C
          A(II,JJ)=1.0D 00
          IF (II.EQ.K)GO TO 60
C
C         SWITCH THE KTH ROW AND THE II TH ROW
```

```
C
            D=-D
           DO 5 J=K,N
            C=A(K,J)
            A(K,J)=A(II,J)
    5       A(II,J)=C
            C=B(K)
            B(K)=B(II)
            B(II)=C
   60       IF (JJ.EQ.K)GO TO 70
C
C            STORE THE LOCATION OF THE MAX PIVOT
C
            II=JPRM(JJ)
            JPRM(JJ)=JPRM(K)
            JPRM(K)=II
C
C            SWITCH THE KTH AND THE JJTH COLUMNS
C
            D=-D
            DO 6 I=1,N
            C=A(I,K)
            A(I,K)=A(I,JJ)
    6       A(I,JJ)=C
   70       IF(K.EQ.N)GO TO 1
            DO 7 I=KPO,N
            DO 8 J=KPO,N
    8       A(I,J)=A(I,J)-A(I,K)*A(K,J)
            B(I)=B(I)-A(I,K)*B(K)
    7       A(I,K)=0.0D 00
    1       CONTINUE
            END
```

SUBROUTINE: DI3

```
      SUBROUTINE DI3(BAM,ANS)
C
C     THIS SUBROUTINE COMPUTES 2 OPTIONS.
C     OPTION 1 IS SOLID-FLUID-RIGID BOUNDARIES
C     OPTION 2 IS SOLID-FLUID-FREE BOUNDARIES
C
      COMMON AKA,AKDA,BAMS,ACC,COCD,BDA,RHM,KB
      IF (BAM) 31,32,32
31    AKA=0.
      GO TO 33
32    AKA=AKDA/BAM
33    AKSA=AKDA/BAMS
      Q2A=AKA**2-AKDA**2
      S2A=AKA**2-AKSA**2
      IF (RHM) 65,65,40
40    AKFA=AKDA/COCD
      QF2A=AKA**2-AKFA**2
      IF (QF2A) 61,60,60
60    QFA=SQRT(QF2A)
      QFB=BDA*QFA
C     COMPUTES BESSEL FUNCTIONS
      CALL BESK(QFA,0,YOQ,IER)
      CALL BESK(QFA,1,Y1Q,IER)
      CALL BESK(QFB,0,YOQB,IER)
      CALL BESK(QFB,1,Y1QB,IER)
      CALL IO(QFA,BFOQ)
      CALL IO(QFB,BFOQB)
      CALL INUE(QFA,1,BFOQ,BF1Q)
      CALL INUE(QFB,1,BFOQB,B1QB)
      QFA=-QFA
      QFB=-QFB
      YOQ=-YOQ
      YOQB=-YOQB
      GO TO 65
61    QFA=SQRT(-QF2A)
      QFB=BDA*QFA
      CALL BESJ(QFA,0,BFOQ,ACC,IER)
      CALL BESJ(QFA,1,BF1Q,ACC,IER)
      CALL BESJ(QFB,0,BFOQB,ACC,IER)
      CALL BESJ(QFB,1,B1QB,ACC,IER)
      CALL BESY(QFA,0,YOQ,IER)
      CALL BESY(QFA,1,Y1Q,IER)
      CALL BESY(QFB,0,YOQB,IER)
      CALL BESY(QFB,1,Y1QB,IER)
65    IF(Q2A)81,80,80
80    QA=SQRT(Q2A)
      CALL IO(QA,BOQ)
      CALL INUE(QA,1,BOQ,B1Q)
      QA=-QA
      GO TO 91
81    QA=SQRT(-Q2A)
      CALL BESJ(QA,0,BOQ,ACC,IER)
D     WRITE (5,85) IER
85    FORMAT (' IER =: ',I2)
      CALL BESJ(QA,1,B1Q,ACC,IER)
```

```
D          WRITE (5,85) IER
91         IF (S2A) 71,70,70
70         SA=SQRT(S2A)
           CALL IO(SA,BOS)
           CALL INUE(SA,1,BOS,B1S)
           GO TO 101
71         SA=SQRT(-S2A)
           CALL BESJ(SA,0,BOS,ACC,IER)
D          WRITE (5,85) IER
           CALL BESJ(SA,1,B1S,ACC,IER)
D          WRITE (5,85) IER
101        D11=BOQ*(2.*AKA**2-AKSA**2)+B1Q*2.*QA
           D12=2.*AKA*(B1S-SA*BOS)
           D21=2.*AKA*QA*B1Q
           D22=B1S*(2.*AKA**2-AKSA**2)
           IF (RHM) 21,21,22
21         ANS=D11*D22-D12*D21
           GO TO 11
22         D13=AKSA**2*RHM*BFOQ
           D14=AKSA**2*RHM*YOQ
           D31=QA*B1Q
           D32=AKA*B1S
           D33=-QFA*BF1Q
           D34=-QFA*Y1Q
           IF (KB-1) 75,75,76
76         D43=BFOQB
           D44=YOQB
           GO TO 78
75         D43=QFB*B1QB
           D44=QFB*Y1QB
78         P12=D11*D22-D12*D21
           P34=D33*D44-D34*D43
           P23=D21*D32-D22*D31
           P14=D13*D44-D14*D43
           ANS=P12*P34+P23*P14
11         RETURN
           END
```

SUBROUTINE: CALIB

```
            SUBROUTINE CALIB(BAM,ANS)
C           THIS SUBROUTINE CORRESFONDS TO THE OPTIONS 3,4, AND 5.
C           OPTION 3 IS WHERE FLUID SOLID AND FREE BOUNDERIES IS ANALYZED
C           OPTION 4 IS WHERE FLUID-SOLID-RIGID BOUNDERIES(CEMENTED)
C           OPTION 5 IS WHERE FLUID-SOLID-RIGID(NOT CEMENTED)
C             IS ANALYZED.
            DOUBLE PRECISION AA(5,5)
            COMMON AKA,AKDA,BAMS,ACC,COCD,BDA,RHM,KB,AA
            IF(BAM)31,32,32
  31        AKA=0
            GO TO 33
  32        AKA=AKDA/BAM
  33        AKSA=AKDA/BAMS
            Q2A=AKA**2-AKDA**2
            S2A=AKA**2-AKSA**2
            AKFA=AKDA/COCD
            QF2A=AKA**2-AKFA**2
            AKB=AKA*BDA
            AKSB=AKSA*BDA
            Q2B=Q2A*BDA*BDA
            S2B=S2A*BDA*BDA
            IF(QF2A)61,60,60
  60        QFA=SQRT(QF2A)
C           BESSEL FUNCTIONS ARE COMPUTED HERE
            CALL IO(QFA,BFOQ)
            CALL INUE(QFA,1,BFOQ,BF1Q)
            QFA=-QFA
            GO TO 65
  61        QFA=SQRT(-QF2A)
            CALL BESJ(QFA,0,BFOQ,ACC,IER)
            CALL BESJ(QFA,1,BF1Q,ACC,IER)
  65        IF(Q2A)81,80,80
  80        QA=SQRT(Q2A)
            QB=SQRT(Q2B)
            CALL IO(QA,BOQ)
            CALL IO(QB,BBOQ)
            CALL INUE(QA,1,BOQ,B1Q)
            CALL INUE(QB,1,BBOQ,BB1Q)
            CALL BESK(QA,0,BOA,IER)
            CALL BESK(QB,0,BOB,IER)
            CALL BESK(QA,1,B1A,IER)
            CALL BESK(QB,1,B1B,IER)
            QA=-QA
            QB=-QB
            BOA=-BOA
            BOB=-BOB
            GO TO 91
  81        QA=SQRT(-Q2A)
            QB=SQRT(-Q2B)
            CALL BESJ(QA,0,BOQ,ACC,IER)
            CALL BESJ(QB,0,BBOQ,ACC,IER)
            CALL BESJ(QA,1,B1Q,ACC,IER)
            CALL BESJ(QB,1,BB1Q,ACC,IER)
            CALL BESY(QA,0,BOA,IER)
            CALL BESY(QB,0,BOB,IER)
```

```
      CALL BESY(QA,1,B1A,IER)
      CALL BESY(QB,1,B1B,IER)
91    IF(S2A)71,70,70
70    SA=SQRT(S2A)
      SB=SQRT(S2B)
      CALL IO(SA,BOS)
      CALL IO(SB,BBOS)
      CALL INUE(SA,1,BOS,B1S)
      CALL INUE(SB,1,BBOS,BB1S)
      CALL BESK(SA,0,BSA0,IER)
      CALL BESK(SB,0,BSB0,IER)
      CALL BESK(SA,1,BSA1,IER)
75    CALL BESK(SB,1,BSB1,IER)
77    BSA1=-BSA1
      BSB1=-BSB1
      GOTO 101
71    SA=SQRT(-S2A)
      SB=SQRT(-S2B)
      CALL BESJ(SA,0,BOS,ACC,IER)
      CALL BESJ(SB,0,BBOS,ACC,IER)
      CALL BESJ(SA,1,B1S,ACC,IER)
      CALL BESJ(SB,1,BB1S,ACC,IER)
      CALL BESY(SA,0,BSA0,IER)
      CALL BESY(SB,0,BSB0,IER)
      CALL BESY(SA,1,BSA1,IER)
85    CALL BESY(SB,1,BSB1,IER)
87    CONTINUE
C     THE DETERMINANT IS BEING FILLED
101   AA(1,1)=BOQ*(2.*AKA**2-AKSA**2)+2.*(QA*B1Q)
      AA(1,2)=(2.*AKA**2-AKSA**2)*BOA+2.*QA*B1A
      AA(1,3)=2.*AKA*(B1S-SA*BOS)
      AA(1,4)=2.*AKA*(BSA1-SA*BSA0)*SA
      AA(1,5)=RHM*AKSA*AKSA*BFOQ
      AA(2,1)=2.*AKA*QA*B1Q
      AA(2,2)=2.*AKA*QA*B1A
      AA(2,3)=(2.*AKA*AKA-AKSA*AKSA)*B1S
      AA(2,4)=(2.*AKA*AKA-AKSA*AKSA)*BSA1*SA
      AA(2,5)=0.0
      IF(KB .EQ. 3)GO TO 120
      AA(3,1)=BB1Q*QB
      AA(3,2)=B1B*QB
      AA(3,3)=AKB*BB1S
      AA(3,4)=AKB*BSB1*SA
      GO TO 150
120   AA(3,1)=BBOQ*(2.*AKB**2-AKSB**2)+2.*QB*BB1Q
      AA(3,2)=(2.*AKB*AKB-AKSB*AKSB)*BOB+2.*QB*B1B
      AA(3,3)=2.*AKB*(BB1S-SB*BBOS)
      AA(3,4)=2.*AKB*(BSB1-SB*BSB0)*SA
150   AA(3,5)=0.0
      IF(KB .NE. 4)GO TO 160
      AA(4,1)=AKB*BBOQ
      AA(4,2)=AKB*BOB
      AA(4,3)=-SB*BBOS
      AA(4,4)=-SB*BSB0*SA
      GO TO 170
160   AA(4,1)=2.*AKB*QB*BB1Q
      AA(4,2)=2.*AKB*QB*B1B
      AA(4,3)=(2.*AKB*AKB-AKSB*AKSB)*BB1S
      AA(4,4)=(2.*AKB*AKB-AKSB*AKSB)*BSB1*SA
170   AA(4,5)=0.0
      AA(5,1)=QA*B1Q
      AA(5,2)=QA*B1A
      AA(5,3)=AKA*B1S
      AA(5,4)=AKA*BSA1*SA
      AA(5,5)=-QFA*BF1Q
      IF (RHM .EQ. 0.0)AA(5,5)=1.0
      CALL DET(5,AA,ANS)
      RETURN
      END
```

SUBROUTINE: FILE

```
            SUBROUTINE FILE(N,FREQ,RATIO)
            BYTE FILEX(32)
            REAL FREQ(1000),RATIO(1000)
            WRITE(5,10)
      10    FORMAT(/,'$FILE NAME FOR X,Y DATA:     ')
            READ(5,20)LEN,FILEX
      20    FORMAT(Q,32A1)
            FILEX(LEN + 1)=0
            OPEN(UNIT=2,NAME=FILEX,TYPE='NEW')
            DO 40 J=1,N
                    WRITE(2,30)FREQ(J),RATIO(J)
      30            FORMAT(2E20.10)
      40    CONTINUE
            CLOSE(UNIT=2)
            CONTINUE
            WRITE(5,45)
    45      FORMAT(/,' FILE IS STORED IN 1 FILE WITH FORMAT (2E20.0) ')
            RETURN
            END
```

SUBROUTINE: FUNC

```
      SUBROUTINE FUNC(B,IOP,ANS)
      IF(IOP .GE. 3)GO TO 20
      CALL DI3(B,ANS)
      GO TO 50
20    CALL CALIB(B,ANS)
50    RETURN
      END
```

```
RUN WGUIDE <CR>
ENTER # CORRESPONDING TO OPTION DESIRED
1) SAMPLE IN FLUID TUBE WITH RIGID BOUNDARY
2) SAMPLE IN FLUID TUBE WITH FREE BOUNDARY
3) FLUID FILLED ELASTIC TUBE, OR EMPTY TUBE
4) FLUID FILLED COATED RIGID TUBE (CEMENTED)
5) FLUID FILLED COATED RIGID TUBE (NOT CEMENTED)
6) FLUID TUBE WITH RIGID BOUNDARY
7) FLUID TUBE WITH FREE BOUNDARY
3 <CR>

ENTER RATIO OF SHEAR TO BULK MODULUS (G/K)  .279  <CR>
ENTER RATIO OF FLUID TO SOLID SPEED (CO/CD)  .315 <CR>

ENTER RATIO OF FLUID TO SOLID DENSITY
 (FOR EMPTY TUBE SET RHM=0,VACUUM TUBE SET RHM=NEG VALUE)   0  <CR>

SHEAR= .451E+00 YOUNG= .747E+00 RAYLEIGH= .423E+00

ENTER X VALUE  (SET KDA NEGATIVE FOR ASYMPTOTES)  -1  <CR>

ENTER MIN,MAX,STEP  (HORIZONTAL SCAN)  .1,4.,.05  <CR>

ENTER RATIO OF TUBE TO SAMPLE RADIUS (B/A)  1.134 <CR>

ASYMPTOTE AT X =    0.75625 +/- 0.250E-03
NO MORE ASYMPTOTES

ENTER # CORRESPONDING TO CHANGE
1) OPTION
2) SHEAR MODULUS/BULK MODULUS (G/K)
3) SPEED IN FLUID/SPEED IN SOLID (CO/CD)
4) FLUID DENSITY/SOLID DENSITY (RHM)
5) MIN MAX, STEP (Y VALUES)
6) TUBE RADIUS/SAMPLE RADIUS (B/A)
7) X VALUE (KDA)
8) MIN,MAX,STEP (X VALUES)
9) STORE DATA
10) EXIT FROM PROGRAM
8  <CR>

ENTER X VALUES: START(KNOWN),END,STEP(+/-)  .1,2.5,.5  <CR>

ENTER MIN,MAX,STEP  (VERTICAL SCAN)  .1,1.5,.1
X =    0.10000 Y =    0.74605 ERROR = 0.500E-04
X =    0.60000 Y =    0.63565 ERROR = 0.500E-04
X =    1.10000 Y =    0.19835 ERROR = 0.500E-04
X =    1.60000 Y =    0.21645 ERROR = 0.500E-04
X =    2.10000 Y =    0.23655 ERROR = 0.500E-04

ENTER # CORRESPONDING TO CHANGE
1) OPTION
2) SHEAR MODULUS/BULK MODULUS (G/K)
3) SPEED IN FLUID/SPEED IN SOLID (CO/CD)
4) FLUID DENSITY/SOLID DENSITY (RHM)
5) MIN MAX, STEP (Y VALUES)
6) TUBE RADIUS/SAMPLE RADIUS (B/A)
7) X VALUE (KDA)
8) MIN,MAX,STEP (X VALUES)
9) STORE DATA
10) EXIT FROM PROGRAM
9 <CR>
```

```
$FILE NAME FOR X,Y DATA:
DATA <CR>

ENTER # CORRESPONDING TO CHANGE
1) OPTION
2) SHEAR MODULUS/BULK MODULUS (G/K)
3) SPEED IN FLUID/SPEED IN SOLID (CO/CD)
4) FLUID DENSITY/SOLID DENSITY (RHM)
5) MIN MAX, STEP (Y VALUES)
6) TUBE RADIUS/SAMPLE RADIUS (B/A)
7) X VALUE (KDA)
8) MIN,MAX,STEP (X VALUES)
9) STORE DATA
10) EXIT FROM PROGRAM
10 <CR>
```

1) All underlined portions are user supplied.

2) <CR> indicates 'RETURN'

DESCRIPTION OF PROGRAM IMPED

The program IMPED computes values for the complex dimensionless propagation speed in the solid sample (assumed infinite in length) inside the impedance tube. Input to the program are the complex values for the ratio of shear modulus to bulk modulus G/K, and the speed in the fluid divided by the dilatational wave speed in the solid, c_o/c_d. Thus, it may be considered as the complex counterpart to program WGUIDE, Option 1. The root finding technique is identical to that used in program RTUBE.*

Subroutines for Bessel functions were written by direct series expansion since the real part of the argument is small enough that a small number of terms suffices. The names of the subroutines are CBJO, CBJ1, CBYO, and CBY1, where the last two letters of each name indicate kind and order of the Bessel function.

The output produced by the program is in the form of real and imaginary parts of the relative wave speed $\beta = c/c_d$, each with its own error limit, corresponding to the input error limits set separately for the two terms. The attenuation constant may be readily calculated by $\alpha = -k_d\beta_2/(\beta_1{}^2 + \beta_2{}^2)$ where α is the attenuation per unit of length and β_1 and β_2 are the real and imaginary parts of β which is equal to c/c_d. If the propagation of the wave in water is assumed to be without loss (c_o real), the input value of AKDA should have the same ratio of imaginary to real part as COCD.

Important Variables in IMPED

Input Variables

AKDA	($k_d a$), Dimensionless wave number of dilatational waves, $= (\omega a)/c_d$ (real & complex)
AMI	# of iterations
ANR	# of divisions of imaginary part
BAM	(c/c_d) relative wave speed; enter real part of c/c_d (ZNOT)
BDA	(b/a) b = radius of tube a = radius of cylindrical sample
COCD	(c_o/c_d) - wave speed in fluid divided by dilatational wave speed in solid (real and complex)
DELZ	(used in computation of stepsize)
GOK	(G/K) (real and complex) G = shear modulus, K = bulk modulus

* P.S. Dubbelday, "Complex Root-Finding Program with Application to the Dispersion Relation of Waves Propagating in a Fluid Plate," NRL Memorandum Report 4559, November 1981.

RHM	(ρ_o/ρ_s) ratio of density of fluid to density of solid; if RHM < 0 solid is in vacuum
TOL1	Tolerance of real part
TOL2	Tolerance of imaginary part

Computational Variables

AI	i, imaginary unit
AKA	(ka), dimensionless wave number, $= \frac{\omega a}{c}$ a = radius of cylinder ω = angular frequency c = propagation speed
AKDA	($k_d a$) Dimensionless wave number of dilatational waves, $= \omega a/c_d$, c_d = dilatational wave speed
AKFA	$k_o a$ = Dimensionless wave number in fluid
AKSA	($k_s a$), Dimensionless wave number of shear waves $k_s a = \omega a/c_s$
ANR	# of divisions of imaginary part
AMGA	Imaginary part of wave speed β
BAMS	(c_s/c_d), relative wave speed of shear waves
DEL	Real stepsize
DELH, DELV	Step sizes for movement of test pairs, horizontally and vertically.
DELP	Intermediate symbol of stepsize DEL.
DELZ	Δz, used in computation of approximation to $\partial f/\partial z$
DFDZ	Approximation for $\partial f/\partial z$.
DS(Z):	Function subprogram
DZ	Complex stepsize
F1	Function evaluated at Z1: $f(z_1)$
F2	Function evaluated at Z2: $f(z_2)$
F3	Function evaluated at Z3: $f(z_3)$
F4	Function evaluated at Z4: $f(z_4)$

FV1, FV2 FH1, FH2	Flags to indicate direction of motion of test pairs
FSTEP	Factor to adjust step size DEL
IOP	Return option number = 1 Program returns to entering COCD, GOK, AKDA = 2 Program returns to entering Real Seed, # of iterations, DELZ, BDA = 3 Program returns to entering AWR, TOL (Real), TOL (Imag.), RHM = 4 Program exits
KAM	= AMI
KR, KL KUP, KDOWN	Counters in loops to check number of iterations
NR	= ANR
NRV	Counter in advancing the parameter RHO
Q2A	$(qa)^2 = (ka)^2 - (k_d a)^2$
QF2A	$(q_o a)^2 = (ka)^2 - (k_o a)^2$
QF2B	$(q_o b)^2 = (kb)^2 - (k_o a)^2$
REGA	Real part of relative wave speed β
RHM	Nominal density of fluid loading the plate, divided by density of plate material.
RHO	Stepwise varied value of relative density, varying from zero to RHM
SIGN	Expression, the algebraic sign of which determines location of root relative to test pair.
TOL	Limit for relative error in the root.
Z	Variable for root used in calling subroutines
Z1, Z2	Vertical test pair
Z3, Z4	Horizontal test pair
ZNOT	Value for the real root of the dispersion relation from a real root finder, serving as the seed for starting the program.
ZVAR	Intermediate value of root, serving as seed for the next step, in parameter RHO

ZVAR	Intermediate value of root, serving as seed for the next step, in parameter RHO

Output Variables

AMGA	Imaginary part of root
ZEGA	Real part of root

Major Computation Block in IMPED

Refer to major computation blocks in RO819

```
C
C       COMPLEX ROOTFINDING ROUTINE
C       WRITTEN BY TINA RUGGIERO
C       DECEMBER 17,1981
C
C       THIS COMPLEX ROUTINE INTRODUCED BY DUBBELDAY
C       ALLOWS THE USER TO SOLVE FOR BOTH THE REAL AND IMAGINARY
C       PARTS OF A COMPLEX ROOT, GIVEN A REAL ROOT IN THE
C       VICINITY AS A SEED.
C
C
C       SUBROUTINES USED:
C         CDI3 (CONTAIN FUNCTION DESCRIBED BY DETERMINANT OF A SET OF
C              BOUNDARY CONDITIONS)
C         CBJ0 (BESSEL FUNCTION (J0))
C         CBJ1 (BESSEL FUNCTION (J1))
C         CBY0 (BESSEL FUNCTION (Y0))
C         CBY1 (BESSEL FUNCTION (Y1))
C
        COMPLEX Z,DSP,AI,DFDZ,DZ,ZVAR,Z1,Z2,Z3,Z4,F1,F2,F3,F4,
        1 GOK,DSP2,CDI3,CMPX,AKDA,AKA,BAMS,COCD,HGOK,HAKDA,HCOCD
        COMMON AKA,AKDA,COCD,BDA,RHM,GOK
C
C       INPUT SECTION
5       WRITE(5,10)
10      FORMAT(/'$ENTER COCD(R,I),GOK(R,I),AKDA(R,I)   ')
        READ(5,20)HCOCD,HGOK,HAKDA
20      FORMAT(6F10.0)
25      WRITE(5,30)
30      FORMAT(/'$ENTER REAL SEED,# OF ITER.,DELZ,BDA   ')
        READ(5,40)ZNOT,AMI,DELZ,BDA
40      FORMAT(4F10.0)
        KAM=AMI
        BAMS=CSQRT(GOK/(1.+4.*GOK/3.))
45      WRITE(5,50)
50      FORMAT(/'$ENTER ANR,TOL (REAL),TOL(IMAG),RHM   ')
        READ(5,60)ANR,TOL2,TOL1,RHM
60      FORMAT(4F10.0)
        AI=CMPLX(0.,1.0)
        GOK=HGOK
        COCD=HCOCD
        AKDA=HCOCD
        RGOK=REAL(GOK)
        RAKDA=REAL(AKDA)
        YGOK=AIMAG(GOK)
        YAKDA=AIMAG(AKDA)
        RCOCD=REAL(COCD)
        YCOCD=AIMAG(COCD)
        NR=INT(ANR)
        NRV=0
        ZVAR=ZNOT
        GOK=CMPLX(RGOK,0.0)
        AKDA=CMPLX(RAKDA,0.0)
        COCD=CMPLX(RCOCD,0.0)
        Z=ZVAR+DELZ
        DFDZ=CDI3(Z)/DELZ
```

```
      GOK=GOK+AI*YGOK/ANR
      AKDA=AKDA+AI*YAKDA/ANR
      COCD=COCD+AI*YCOCD/ANR
      NRV=NRV+1
      DZ=-CDI3(ZVAR)/DFDZ
      X=ABS(REAL(DZ))
      Y=ABS(AIMAG(DZ))
C
C     DEL1 AND DEL2 ARE HORIZONTAL AND VERTICAL STEP SIZES
      DEL1=Y
      DEL2=X

      WRITE(5,16)DEL1,DEL2
16    FORMAT(/,' DEL1= ',E8.2,'    DEL2= ',E8.2)
      WRITE(5,57)
      TYPE *,'------------------------------------------------------------
      DELP=DEL1
      DELQ=DEL2
C     FIND DIRECTION OF INITIAL VERTICAL PAIR
70    DEL2=DELP
      DEL1=DELQ
64    Z1=ZVAR-AI*DEL1
      Z2=ZVAR+AI*DEL1
65    F1=CDI3(Z1)
      F2=CDI3(Z2)
      KR=0
      KL=0
      KUP=0
      KDOWN=0
C     THE RESULT OF SIGN IS TO DETERMINE DIRECTION OF PAIR
      SIGN=AIMAG(F1)*REAL(F2)-REAL(F1)*AIMAG(F2)
      IF (SIGN) 71,72,73

71    FH1=-1
      GO TO 74
72    TYPE *,'LEFT RIGHT SIGN'
      GO TO 100
73    FH1=1
74    DELH=FH1*DEL1
C     LOOP TO FIND VERTICAL LINES BRACKETING ROOT

75    Z1=Z1+DELH
      Z2=Z2+DELH
      F1=CDI3(Z1)
      F2=CDI3(Z2)
C
C     SIGN IS USED TO DETERMINE DIRECTION PAIR IS MOVED
      SIGN=AIMAG(F1)*REAL(F2)-AIMAG(F2)*REAL(F1)
      IF (SIGN) 81,72,83
81    FH2=-1
      KL=KL+1
      IF(KAM-KL)99,99,84
99    TYPE *,'EXIT LEFT'
      GO TO 100
83    FH2=1
      KR=KR+1
```

```
      IF(KAM-KR)98,98,84
98    TYPE *,'EXIT RIGHT'
      GO TO 100
84    IF(FH1*FH2)92,75,75
C     FIND HORIZONTAL TEST PAIR DIRECTION
92    Z4=(Z1+Z2)/2.+DEL2
      KR=0
      KL=0
      Z3=(Z1+Z2)/2.-DEL2
      F3=CDI3(Z3)
      F4=CDI3(Z4)
      SIGN=AIMAG(F3)*REAL(F4)-AIMAG(F4)*REAL(F3)
      IF(SIGN)101,102,103
101   FV1=1.
      GOTO 104
102   TYPE *,'UP-DOWN SIGN= ZERO'
      GOTO 100
103   FV1=-1
104   DELV=FV1*DEL2
C     LOOP TO FIND HORIZONTAL LINES BRACKETING ROOT
105   Z4=Z4+AI*DELV
      Z3=Z3+AI*DELV
      F3=CDI3(Z3)
      F4=CDI3(Z4)
      SIGN=AIMAG(F3)*REAL(F4)-AIMAG(F4)*REAL(F3)
      IF(SIGN)111,102,113
111   FV2=1.
      KUP=KUP+1
      IF(KAM-KUP)97,97,114
C
97    TYPE *, 'EXIT UP'
      GOTO 100
113   FV2=-1.
      KDOWN=KDOWN+1
      IF(KAM-KDOWN)96,96,114
96    TYPE *, 'EXIT DOWN'
      GO TO 100
114   IF(FV1*FV2)122,105,105
122   REGA=REAL(Z1)
      AMGA=AIMAG(Z3)
      KUP=0
      KDOWN=0
133   AMGA=ABS(AMGA)
      DREG=ABS(REGA-ZNOT)
C
C     CHECK IF REQUIRED PRECISION REACHED
      IF(DEL1/AMGA-TOL1 .LE. 0)GOTO 132
      IF(DEL2/REGA-TOL2 .LE. 0)GO TO 135
      DEL2=DEL2/10.
      GO TO 92
135   DEL1=DEL1/10.
      DEL2=DEL2/10.
      Z1=(Z3+Z4)/2.-AI*DEL1
      Z2=(Z3+Z4)/2.+AI*DEL1
      GOTO 65
132   REGA=REAL(Z1)-DELH/2.
```

```
      AMGA=AIMAG(Z3)-DELV/2.
      DELH=ABS(DELH/2.)
C
      DELV=ABS(DELV/2.)
C
C     OUTPUT SECTION
57    FORMAT(/,'     REAL',7X,'IMAGINARY',5X,'REAL ERROR',5X,'IMAG ERROR'.
      WRITE(5,58)REGA,AMGA,DELH,DELV
58    FORMAT(E10.4,4X,E10.4,4X,E10.4,6X,E10.4)
C
C     CHECK IF FINAL  GOK AND AKDA IS REACHED
      IF(NR-NRV)145,145,146
146   ZVAR=REGA+AMGA*AI
      GOK=GOK+AI*YGOK/ANR
      AKDA=AKDA+AI*YAKDA/ANR
      COCD=COCD+AI*YCOCD/ANR
      NRV=NRV+1
      GOTO 70
145   CONTINUE
      TYPE *, ' '
100   TYPE *,' ENTER OPTION # (1,2,3,4)  '
      READ(5,160)IOP
160   FORMAT(I2)
      GO TO (5,25,45,550),IOP
550   END
```

```
                  FUNCTION CDI3(BAM)
                  COMMON AKA,AKDA,COCD,BDA,RHM,GOK
                  COMPLEX AKA,AKDA,BAMS,COCD,AKSA,Q2A,S2A,AKFA,QF2A,QFA,QFB,BFOQ,
                  1 BF1Q,B1QB,YOQ,Y1Q,Y1QB,QA,BOQ,B1Q,SA,BOS,B1S,D11,D12,D21,D22,
                  1 DI3,D14,D31,D32,D33,D34,D43,D44,P12,P34,P23,P14,CDI3,QP2A,AI,
                  1 QPF2A,SP2A,QPFA,BAM,D13,CSQRT,GOK
                  BAMS=CSQRT(GOK/(1.+4.*GOK/3.))
                  AI=CMPLX(0.0,1.0)
                  AKA=AKDA/BAM
                  AKSA=AKDA/BAMS
                  QP2A=AKDA**2-AKA**2
                  SP2A=AKSA**2-AKA**2
                  IF(RHM)65,65,40
        40        AKFA=AKDA/COCD
                  QPF2A=AKFA**2-AKA**2
                  IF(REAL(QPF2A))45,46,46
        46        QPFA=CSQRT(QPF2A)
                  GO TO 50
        45        QPFA=-AI*CSQRT(-QPF2A)
        50        QFB=BDA*QPFA
                  IF(ABS(QFB) .GT. 10.0)GOTO 1000
                  CALL CBJO(QPFA,BFOQ)
                  CALL CBJ1(QPFA,BF1Q)
                  CALL CBJ1(QFB,B1QB)
                  CALL CBYO(QPFA,YOQ)
                  CALL CBY1(QPFA,Y1Q)
                  CALL CBY1(QFB,Y1QB)
        65        IF(REAL(QP2A))81,80,80
        80        QA=CSQRT(QP2A)
                  GO TO 85
        81        QA=-AI*CSQRT(-QP2A)
        85        CALL CBJO(QA,BOQ)
                  CALL CBJ1(QA,B1Q)
                  IF(REAL(SP2A))71,70,70
        70        SA=CSQRT(SP2A)
                  GO TO 90
        71        SA=-AI*CSQRT(-SP2A)
        90        CALL CBJO(SA,BOS)
                  CALL CBJ1(SA,B1S)
                  D11=BOQ*(2.*AKA**2-AKSA**2)+B1Q*2.*QA
                  D12=2.*AKA*(B1S-SA*BOS)
                  D21=2.*AKA*QA*B1Q
                  D22=B1S*(2.*AKA**2-AKSA**2)
                  IF (RHM)21,21,22
        21        CDI3=D11*D22-D12*D21
                  GO TO 11
        22        D13=AKSA**2*RHM*BFOQ
                  D14=AKSA**2*RHM*YOQ
                  D31=QA*B1Q
                  D32=AKA*B1S
                  D33=-QPFA*BF1Q
                  D34=-QPFA*Y1Q
                  D43=QFB*B1QB
                  D44=QFB*Y1QB
                  P12=D11*D22-D12*D21
                  P34=D33*D44-D34*D43
                  P23=D21*D32-D22*D31
                  P14=D13*D44-D14*D43
                  CDI3=P12*P34+P23*P14
        11        RETURN
        1000      TYPE *,'ARGUMENT FOR BESSEL FUNCTIONS, TOO LARGE'
        2000      END
```

```
      SUBROUTINE CBJ1(Z,ANS)
C
C     BESSEL FUNCTION J1
C     (COMPUTED BY SERIES)
C
      COMPLEX Z,FUNC,ANS,CMPLX,FIRST
      CALL ERRSET(73,,.FALSE.,,.FALSE.,,)
      TOL=.000001
    8 ERROR=1.0
      FUNC=Z/2
      ANS=Z/2
      RHO=CABS(Z)
      DO 30 R=2,1000
      FIRST=-1.0*FUNC*(Z/2.0)**2/(R*R)
      FUNC=FIRST*2.0*R/(2*R-2.0)
      ANS=ANS+FUNC
      ERROR=ERROR*(RHO/2.0)**2/(R*R)
      IF(ERROR/CABS(ANS)-TOL)50,50,30
   30 CONTINUE
   50 RETURN
      END
```

```
      SUBROUTINE CBJO(Z,ANS)
>
C
C     BESSEL FUNCTION JO
C     (COMPUTED BY SERIES)
C
      COMPLEX Z,FUNC,ANS,CMPLX
8     ANS=CMPLX(1.0,0.0)
      TOL=.000001
      ERROR=1.0
      FUNC=CMPLX(1.0,0.0)
      RHO=CABS(Z)
      DO 30 R=1,1000
      FUNC=(-1.0)*FUNC*(Z/2.0)**2/(R*R)
      ANS=ANS+FUNC
      IF (CABS(ANS) .EQ. 0.0)GO TO 30
      ERROR=ERROR*(RHO/2.0)**2/(R*R)
      IF(ERROR/SQRT(REAL(ANS)**2+AIMAG(ANS)**2)-TOL)50,50,30
30    CONTINUE
50    RETURN
      END
```

```
RUN IMPED

ENTER COCD(R,I),GOK(R,I),AKDA(R,I)  .2358,.002358,.4541,.004541,.2,.002  <CR>

ENTER REAL SEED,# OF ITER.,DELZ,BDA  .97875,200,.01,1.001 <CR>

ENTER ANR,TOL (REAL),TOL(IMAG),RHM  3,.01,.001,.12805  <CR>

DEL1= 0.24E-03     DEL2= 0.16E-05

   REAL        IMAGINARY      REAL ERROR      IMAG ERROR

-----------------------------------------------------------

.9788E+00     0.2313E-03     0.8249E-07      0.1217E-04

.9788E+00     0.4870E-03     0.8249E-07      0.1217E-04

.9788E+00     0.7183E-03     0.8249E-07      0.1217E-04

 ENTER OPTION # (1,2,3,4)
 4  <CR>
```

1) All underlined portions are user supplied.

2) <CR> indicates 'RETURN'

APPENDIX B
Correction for Imperfect Reflector in Impedance Tube

The analysis of the role of the reflector in the impedance tube starts from the observation that there are traveling waves in both directions in the water of the impedance tube, in the sample, and in the reflector, each with its own amplitude and phase. By equating stress and particle velocity at each of the interfaces one may infer the reflection coefficient for the following cases: r_{sa} is the reflection coefficient without the sample in place, r_{st} is the reflection coefficient without the sample. One would like to relate these measured quantities to the reflection coefficient r used in Eq. (2), which implies a perfect reflector. The length ℓ of the reflector is chosen such that $k_r\ell$ is close to $\pi/2$. The subscript r refers to the reflector. Define δ by $k_r\ell = \pi/2 + \delta$, δ is a small complex number. One finds that

$$r_{sa} = \frac{1 - i(\tan k_s d)(\rho c)_o/(\rho c)_s - i\varepsilon[1 - i(\tan k_s d)(\rho c)_s/(\rho c)_o]}{1 + i(\tan k_s d)(\rho c)_o/(\rho c)_s + i\varepsilon[1 + i(\tan kd)(\rho c)_s/(\rho c)_o]} , \qquad \text{(B1)}$$

$$r_{st} = e^{-2ik_o d}\,\frac{1 - i\varepsilon}{1 + i\varepsilon} , \qquad \text{(B2)}$$

$$r = \frac{1 - i(\tan k_s d)(\rho c)_o/(\rho c)_s}{1 + i(\tan k_s d)(\rho c)_o/(\rho c)_s} . \qquad \text{(B3)}$$

where the parameter ε is defined by $\varepsilon = (\tan \delta)(\rho c)_o/(\rho c)_r$.

In the present data analysis, the reflection coefficient r in Eq. (2) is replaced by r_c, defined by

$$r_c = \exp(-2ik_o d)r_{sa}/r_{st} . \qquad \text{(B4)}$$

The correction factor applied to r_{sa} according to this equation may be expressed in terms of the small parameter ε by

$$\exp(-2ik_o d)/r_{st} \approx 1 + 2i\varepsilon . \qquad \text{(B5)}$$

The proper correction factor is r/r_{sa}, which may be expressed to the same order by

$$r/r_{sa} \approx 1 + 2iF\varepsilon \quad , \tag{B6}$$

where the factor F is given by

$$F = 1/\left\{\cos^2 kd + \sin^2 kd[(\rho c)_o/(\rho c)_s]^2\right\} . \tag{B7}$$

This factor F may be noticeably different from one, if the complex specific acoustic impedance of the sample material is not very close to that of the water in the tube. Even though densities and propagation speeds of sample and water might be close, as is the case for elastomers, a difference in attenuation still would make this factor unequal to one.

This proposed correction does not consider the effect of the termination of the tube by the reflector and the consequent deviation from the idealized situation of an infinite waveguide.

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